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A SYSTEMATICALLY VARYING PERIOD WITH AN AVERAGE LENGTH OF 28 MONTHS IN WEATHER AND SOLAR PHENOMENA [1]

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SYNOPSIS

During the last 25 years a number of writers have called attention to a short cycle in weather elements and have given estimates of its mean length varying from 2.5 to 3.5 years. The diversity in these estimates is due in part to the practice of some investigators of using data with a time interval of 12 months, which leads to high values of the intervals; others employed consecutive 12-month means and derived values of 2.5 to 3.0 years.

In the present paper various statistical criteria and methods are employed to show the extent to which a given succession of meteorological data, such as yearly means of temperature, differs from a purely fortuitous sequence of similar data. In high latitudes the mean interval between two consecutive maxima or minima is less than 3.0 years, which is the mean interval for a perfectly fortuitous sequence. This and other criteria indicate a systematic and persistent tendency to a recurrence of similar phases which differs from that due to chance alone. and this period of recurrence is approximately 2.0 to 2.5 years.

Two methods of investigation, the empirical and the analytical, are discussed and contrasted. The empirical method is illustrated by Wolfer's determination of the epochs of maxima and minima of the solar spots, by Brückner's determination of the epochs of the 35-year cycle, and by Wallén's investigation of the 2.5-year cycle in temperature, rainfall, and lake levels in Sweden.

It is maintained that analytical methods applied to meteorological data, illustrated by Brunt's periodogram analysis of Greenwich temperatures and Beveridge's periodogram of wheat prices in Europe for 300 years, have led to negative results in that no stable periodicity of uniform length has been certainly shown to exist.

In the present investigation two 12-month means per year have been employed, one centered on January 1 and the other on July 1. The adequacy of data in this form for disclosing satisfactorily a cycle of 2.0 to 2.5 years in length is shown by both empirical and analytical methods.

Contemporaneous curves showing meteorological conditions for a number of stations have been drawn for the regions studied, viz. northern Europe and northern United States.

By careful scrutiny and comparison of these curves the epochs of maxima and minima of the short cycle have been determined for pressure and temperature in Europe since 1740 and for temperature in the United States since 1780. Similar phases of pressure over southwestern Europe and of temperature over northern Europe and the northern United States are nearly coincident; the deviations from coincidence are shown to correspond to the accidental errors of observation. Epochs of maxima and minima have also been derived for the pressure in Greenland, which are in general opposite to those in southwestern Europe. The epochs of pressure and temperature for Portland, Oreg., are shown to differ markedly in phase from those for stations in the upper Mississippi Valley. The epochs of maximum and minimum storm velocity in the United States and of maximum and minimum interdiurnal variability of pressure at St. Louis have been derived and it is found that when the temperature over the northern boundary of the United States is low, storm areas move rapidly and the day-to-day fluctuations in pressure and temperature at St. Louis are large. Variations in temperature at New Orleans occur later than at St. Paul, the average difference in the epochs of the short cycle being about four months.

The variations of rainfall show the short cycle with less regularity than those of pressure and temperature, in conformity with the more nearly fortuitous character of this element. In general the epochs of maximum rainfall over northern Europe occur near the epochs of minimum pressure over southern Europe.

It is found that the period under discussion has a mean length of about 28 months, subject to systematic variations in length attributed to the 11-year sunspot period, the 35-year Brückner variation, and to a long secular change, perhaps indicative of a 300-year cycle. The latter is indicated by the general increase in length from about 2.0 years at the middle of the eighteenth century to about 2.5 years at the present time. The 35-year cycle is suggested by a shortening of the period at or near the wet epochs, and a corresponding lengthening at the dry epochs of the Brückner cycle. The 11-year variation seems to be shown by a secondary decrease in the length a few years after the epochs of maxima of sun spots.

A 28-month periodicity is shown to exist in the variations of the mean latitude of sun spots, which shifts systematically from one hemisphere to the other over a range of about 10°. The 11-year and the 35-year variations are shown also to exist in the length of this solar period.

A fairly large correlation exists between the latitude of spots, (regarding north latitude as positive and south latitude as negative), and the temperature at St. Paul one year later, the coefficient being −.56.

Wolfer's smoothed sun spot relative numbers since 1750, with the 11-year variation eliminated, disclose secondary maxima with a tendency to recurrence every 2.3 years, as an average. The 11 and 35 year, as well as the long secular variations are evident in the recurrence of these maxima.

A new graphical scheme is described which facilitates the accurate evaluation of the mean length of the period at any time.

OUTLINE OF TOPICS

[1] The substance of this paper was published in Mo. WEATHER REV., January, 1924, 52 : 38. The writer desires to express his appreciation of the suggestions and criticisms offered by Prof. C. F. Marvin during the final revision.

VI. THE 28-MONTH PERIOD IN SOLAR PHENOMENA.
Variations in mean latitude of sun spots.
Variations in Wolfer's relative sun spot numbers.
VII. CORRELATION OF SOLAR AND METEOROLOGICAL DATA.
VIII. GRAPHICAL EVALUATION OF THE LENGTH OF A VARIABLE PERIOD.
IX. CONCLUSION.

I. EARLIER INVESTIGATIONS OF A PERIOD ABOUT THREE YEARS IN LENGTH

The existence of a period in weather phenomena, variously estimated at 2.5 to 3.5 years in length, has been affirmed by a number of students of weather variations during the past 25 years. A very comprehensive summary of such investigations, with bibliography, is contained in the paper of Helland-Hansen and Nansen,[2] and a careful study of this paper should convince an impartial reader that there is something here deserving further investigation.

Probably the first to call attention to this short period was Bigelow,[3] in 1898. He published annual values for 15 years of pressure and temperature for stations in the northwestern United States, together with average velocities of low-pressure areas and cold waves in latitude and longitude, with a plot of the data. He states: "Indeed the occurrence of four subordinate crests in the 11-year period suggests strongly that a 2¾-year period is superimposed upon the long sweep of the periodic curve. Apparently this is more at the basis of the seasonal variations in the United States than anything else, so that in long-range forecasting this period must be very carefully considered." In 1902[4] and 1903 he published curves of pressure and temperature variations over the entire globe and pointed out that a period of about 3 years is shown by the curves.

The Lockyers[5] published curves of pressure and rainfall variations in India; also curves of solar phenomena, and stated that a short period of about 3.5 years was shown by them. In 1903 they extended the investigation to cover the entire globe and confirmed the earlier results.

Braak[6] in 1910 found a 3.5-year period in the meteorological elements at Batavia. In later years he referred to this as a 3-year period. Variations in temperature occur a few months later than corresponding variations in pressure.

Arctowski[7] has published several papers with curves of consecutive 12-month means relating to this short period, which he regarded as 2¾ years in length.

Wallén since 1910 has discussed in numerous papers a short-period variation in the temperature and rainfall of Sweden and the levels of the Swedish lakes Malar and Wener. He finds a period of about 2.5 years. His studies will be analyzed with some fullness in Section IV.

Helland-Hansen and Nansen in their paper (loc. cit.) published curves of monthly means and consecutive 12-month means of various meteorological elements at Batavia, and concluded that a period of 32 to 33 months is indicated by the curves.

Diversity in estimates of length of period and reasons therefor.—It is obvious that there is considerable diversity of opinion regarding the length of the short cycle. Estimates range from 2.5 to 3.5 years. This is wholly accounted for by the fact that those who incline to the longer interval have employed the calendar year mean, which in the case of a 2.5-year period is entirely too large a unit, and the fluctuations of short length and small amplitude are thereby obscured, resulting in a mean length somewhat too large. Bigelow and the Lockyers used yearly means. Others, as Arctowski, Wallén, and Helland-Hansen and Nansen, employed consecutive 12-month means and determined the length of the period to be 2½ to 2¾ years.

II. STATISTICAL EVIDENCE IN FAVOR OF A 2 TO 2½ YEAR PERIOD

Statistical criteria applied to meteorological data.—The writer,[8] in a statistical analysis of meteorological data, has given evidence showing a predominance of the 3-year interval in any series of annual means for most regions of the globe. However, in applying a criterion based on the relative number of changes of sign in a series of yearly departures such as Mielke's temperature departures for 25 districts over the entire globe, there were disclosed wide variations in different regions. The number of changes of sign is 50 per cent of the total number of values for a wholly fortuitous sequence.[9] Actually there were 12 districts with percentages below 50 per cent and 11 districts with 50 per cent or above. The districts below 50 per cent are in the Southern Hemisphere and the lower latitudes of the Northern Hemisphere, including most of the United States. The districts above 50 per cent include northern Europe and Russia. Scandinavia shows an extreme of 60 per cent. A quotation from the paper follows (p. 129):

> The excessively high values in northern Europe and northwest Russia illustrate the extreme variability of weather in high latitudes. It is obvious that a marked deviation either above or below 50 per cent is indicative of a systematic tendency in the variations. These results are interesting as showing how different are the characteristics of meteorological variations in different latitudes and how unsafe it is to draw general conclusions from investigations covering a restricted area.

A value around 50 per cent indicates that the data are nearly fortuitous in their sequence and that the frequency of the 2-year, 3-year, and 4-year intervals is nearly that of unrelated numbers. A value greater than 50 per cent indicates an abnormal excess of the 2-year interval, and a value less than 50 per cent indicates an excess of the 4-year interval.

To illustrate the excessive predominance of the 2-year interval in high latitudes, the yearly mean temperatures at Stockholm from 1757 to 1918 were examined with the following result: The frequency of the 2-year, 3-year, 4-year, etc., intervals expressed as a percentage of the whole number of intervals between successive maxima or minima, are shown in the table below. The bottom row gives the percentage for fortuitous numbers, as derived from Besson's[10] formula.

Interval in years	2	3	4	5	6	7	8
Yearly temperatures, per cent	48	27	21	3	0	0	1
Fortuitous numbers, per cent	40	33	17	7	2	1	0

The excess of the 2-year interval is clearly indicated.

[2] Temperature Variations in the North Atlantic Ocean and in the Atmosphere. Smith. Misc. Coll., Vol. 70, No. 4, 1920.
[3] Weather Bureau Bulletin No. 21, Abstract of a Report on Solar and Terrestrial Magnetism, 1898.
[4] Bigelow, F. H., A Contribution to Cosmical Meteorology. Mo. WEATHER REV. 30: 347. 1902.
[5] Lockyer, J. N., and W. J. S., "On some phenomena which suggest a short period of solar and meteorological changes." Proc. Roy. Soc., 70: 500, 1902.
[6] Braak, C., "Periodische Schwankungen." Met. Zeit. 27: 121, 1910. See also "Atmospheric variations of short and long duration in the Malay Archipelago and eighboring regions, and the possibility to forecast them." Knk. Mag. Met. Obs.o te Batav. Verh. No. 5. 1919.
[7] Arctowski, Henryk, "The solar constant and the variation of atmospheric temperature at Arequipa and some other stations." Bull. Am. Geog. Soc., vol. 44: 598, 1912.
[8] Clough, H. W., "A statistical comparison of meteorological data with data of random occurrence." Mo. WEATHER REVIEW, 49: 128, 1921.
[9] Clough, loc. cit., p. 125.
[10] Besson, Louis, "On the comparison of meteorological data with results of chance." (Translation by E. W. Woolard.) Mo. WEATHER REVIEW, 48: 89, 1920.

The number of maximum or minimum values of a long sequence of fortuitous numbers is one-third the total number of values, so that the average interval between like phases is the interval between three numbers. In the case of the yearly numbers representing the temperature for Stockholm during the period from 1757 to 1794 the mean interval is 2.47 years, while from 1757 to 1826 it is 2.65 years and from 1838 to 1918, 2.82 years. For Copenhagen, 1798 to 1921, the average is 2.73 years. For Halmsted, 1859 to 1918, the average is 2.56 years. For Christiania, 1874 to 1922, the average is 2.61 years. These values, markedly below the theoretical 3.0 for unrelated numbers, are obvious evidence that the variations of the data are systematic, that is, not fortuitous, and indicate a periodicity around 2½ years, or less.

The tendency of the 3-year interval to predominate in lower latitudes is shown by the yearly temperatures at Berlin and Turin. At the former, from 1719 to 1907, the mean interval is almost exactly 3.0 years, while in still lower latitudes, at Turin, the mean interval is 3.5 years. This is due to the systematic decrease in amplitude with decrease in latitude, resulting in the disappearance of fluctuations of small amplitude, when yearly means only are employed.

Another criterion is furnished by the number of times when maxima and minima are separated by an interval of only 1 year, expressed as a percentage of the cases of record. A number of long temperature records in northern Europe were examined and percentages in excess of that for unrelated numbers, 41 per cent (cf. Besson, loc. cit.), was found, as follows: Stockholm, 1757–1826, 53 per cent, 1849–1918, 52 per cent; Halmsted, 1859–1918, 60 per cent; Copenhagen, 1798–1850, 48 per cent; 1850–1921, 59 per cent. These values further illustrate a tendency to an abnormal excess of cases when one extreme followed the opposite extreme with an interval of only 1 year.

In low latitudes, e. g., Turin, the percentage of single rises and falls, 25 per cent, is less than the theoretical percentage for unrelated numbers.

The number of single rises and falls during successive 5-year periods from 1755 was determined for four regions in Europe, the data being based on Köppen's and Mielke's regional departures of temperature. The regions are (1) Great Britain, (2) northern Germany and Holland, (3) west-central Europe, and (4) Austria. The number per lustrum for the four regions named are given in the following table, together with their average. There are well-defined maxima in the lustra 1781–1785, 1811–1815, 1841–1845, 1881–1885, and there are also indications of a maximum around 1910 or 1915 shown by data at Stockholm and Copenhagen. These epochs of maximum single rises and falls per lustrum are nearly synchronous with the Brückner epochs of maximum precipitation 1775, 1815, 1846–1850, 1880, 1915. There is thus shown statistically a tendency for the occurrence of more rapid oscillations of temperature approximating a 2-year period, during the wet portions of the Brückner period. This is consistent with results derived in Section V of this paper.

Table of number of single rises and falls per 5 years:

Lustrum ending	1760	1765	1770	1775	1780	1785	1790	1795	1800	1805	1810	1815	1820	1825	1830
Region 1	----	----	----	1	2	5	2	3	1	2	4	4	1	1	3
" 2	3	3	2	2	1	4	0	1	1	1	3	5	2	3	2
" 3	1	1	0	1	2	4	0	1	1	1	3	5	2	2	2
" 4	----	----	----	2	1	4	1	1	1	1	2	3	1	3	1
Average	2.0	2.0	1.0	1.5	1.5	[1]4.2	0.8	1.5	1.0	1.2	3.0	[1]4.2	1.5	2.2	2.0

[1] Maximum values nearly coincident with wet epochs of Brückner period.

Lustrum ending	1835	1840	1845	1850	1855	1860	1865	1870	1875	1880	1885	1890	1895	1900	1905	1910
Region 1	0	2	3	3	3	3	2	1	1	2	5	2	1	0	0	4
" 2	1	1	5	2	2	3	1	2	1	1	5	2	0	5	2	4
" 3	1	2	4	3	3	3	1	2	1	1	5	5	0	1	2	2
" 4	1	3	5	3	0	2	2	2	1	1	4	0	4	2	1	1
Average	0.8	2.0	[1]4.2	2.8	2.0	2.7	1.5	1.8	1.0	1.2	[1]4.8	2.3	1.2	2.0	1.2	[1]3.0

[1] Maximum values nearly coincident with wet epochs of Brückner period.

A statistical analysis of the composite European temperature departures referred to in section V, from 1730 to 1920 by 50-year periods, (last period 40 years) showing the frequency of the various intervals between consecutive maxima or consecutive minima, expressed as a percentage of the whole number of such intervals, is given in the table below. The data being 6-month means smoothed by the formula $(a+b)\div 2$, are compared with like results from unrelated numbers smoothed by the same formula. The frequencies for unrelated numbers were determined empirically by the writer.

Interval	2	3	4	5	6	7	8	9	10	Sum of frequencies 5 to 10
Unrelated numbers	15	33	22	14	8	5	2	----	----	29
1730–1780	9	36	32	13	9	----	----	----	----	22
1780–1830	2	30	33	9	5	9	9	----	2	34
1830–1880	2	29	32	29	4	4	----	----	----	37
1880–1920	0	36	33	6	17	8	----	----	----	31
Average 1730–1920	4	33	32	14	9	5	2	----	----	30

The significant features are the relatively infrequent occurrence in each of the 50-year periods of the two-interval, and the excess of the four-interval, as compared with the theoretical values for unrelated numbers. Each set of values is based on 100 items, two for each of the 50 years. The approximate mode of the frequency distribution for the average is 3.5, as compared with a mode of about 3.15 for smoothed unrelated numbers.

These results indicate the persistent tendency over a long period of time for meteorological data to show systematic departures from a fortuitous order of succession. The excess of the four-interval, corresponding to a 2-year sequence, points to the existence of a period somewhere near this length of time. The sums of the frequencies for the intervals greater than four show a systematic increase from the early to the later series, while the two-interval shows a corresponding persistent decrease, both of which features are consistent with results deduced in a later section of this paper relative to a secular increase in the length of this period during the last 175 years.

Some persistent biennial recurrences discussed and analyzed.—C. F. Brooks [11] has called attention to sequences of alternately cold and warm winters, or where a winter warmer (or colder) than the preceding winter was followed by a colder (or warmer) winter. Such alternations occurred several times in succession in two or three instances in the nineteenth century, notably 1804–1810, and 1872–1883, and again around 1920. In a discussion of Brooks's paper (p. 73, loc. cit.) I showed that the number of such alternations occurred at three stations with records of 85 to 100 years on an average of 15 per cent in excess of the number required by theory if there

[11] Brooks, C. F., "Sequence of winters in the northeastern United States." MO. WEATHER REV. 49: 71, 1921.

were no relation between the character of successive winters, and furthermore that the particular sequence of 10 to 12 alternations in the seventies and eighties of the last century would occur only once in about 3,000 years, if the occurrence were purely fortuitous. If the remarkable approach to uniformity in amplitude of the series were taken into account, it is probable that a series of 10,000 values would be required to yield such a sequence. It is obvious, therefore, that the existence of such a sequence is strong presumptive evidence that there is a tendency to a cyclical recurrence, at times, in winter temperatures, averaging two years in length.

Reference should here be made to a paper by Clayton [12], in which he discussed a cyclical recurrence of temperature, pressure, and rainfall in the United States, averaging about 25 months, in the late seventies and early eighties of the last century. The regularity of recurrence, however, disappeared in later years.

Douglas [13] notes a prominent 2-year oscillation in tree growth as shown by the rings. He writes:

> In the cross-identification of the trees a constantly recurring feature has been a marked alternation in size of successive rings, giving them the appearance of being arranged in pairs. In the plotted curves this produces a zigzag or seesaw effect. Usually such effect lasts a few years and then disappears, but one example shows unusual persistence—between 750 B. C. and 660 B. C. The even dates show less growth than the odd almost continuously for 60 years, but for the next 30 years the reverse is the case. This is evidently due to a short period of about 2 years in length. It has not been fully studied, but it is prominent in the European groups and in the Vermont group. It frequently shows a duration of a little less than 7 years in one phase with odd dates greater in growth than even dates, and then for the next 7 years reverses its phase. This 14-year cycle is the series of beats the 2-year cycle produces by interfering with the exact annual and biennial effects in the tree. Hence by a simple process its length is found to be in effect frequently 21 or 28 months. Comparison has been made with the rainfall records near the Vermont group and a variable period has been found averaging near the larger value.

This is another instance of a biennial recurrence in much greater frequency than if the climates of successive years were wholly unrelated.

Length of period derived approximately by method of correlation.—The method of correlation affords a ready means of evaluating the degree of relationship shown by two curves. Indirectly it may confirm the existence of a periodic recurrence of values in the data composing a single curve and which may be obvious on simple inspection. For example, if a series of data such as 12-month means at 6-month intervals be correlated with the same data 6, 12, 18, etc., months later, and the process be repeated indefinitely, a recurrent period in the data will be shown by a systematic change in the correlation coefficients from +1.00 to minus values and back to plus values repeatedly as the two curves are by this process in effect shifted so as to bring like phases of the period successively into conjunction or opposition with each other. If the data represent a true periodic function, the coefficients will range between +1.00 and −1.00. A lesser degree of regularity in the recurrence will be shown by lower coefficients. The mean period-length is derived from the average interval between like phases in the series of coefficients. Illustration of the results obtained by this process is shown in the following table, which gives the coefficients for the pressure and temperature variations at St. Paul from 1872 to 1924.

Years	Pressure	Temperature	Years	Pressure	Temperature	Years	Pressure	Temperature
	r	r		r	r		r	r
0.0	+1.00	+1.00	3.0	−0.11	−0.35	6.0	−0.45	−0.11
0.5	+.41	+.36	3.5	−.43	−.46	6.5	−.38	−.22
1.0	−.70	−.67	4.0	−.39	+.19	7.0	−.43	−.34
1.5	−.53	−.47	4.5	+.31	+.35	7.5	+.50	−.00
2.0	+.32	+.38	5.0	+.45	+.25	8.0	−.19	+.44
2.5	+.43	+.40	5.5	+.16	−.00			

The mean length of period to the nearest half year by this method for both pressure and temperature is seen to be 2.5 years. In reality the result by this method probably coincides closely with the mode obtained by a frequency classification of the intervals between the more pronounced maxima and minima. The distribution of phase-intervals for both solar and terrestrial periods appears to be slightly unsymmetrical and the skewness is positive, that is, the arithmetic mean is greater than the mode. Hence the result by the correlation method may be somewhat less than the arithmetic mean.

The method of correlation employed is that of concurrent variations, which gives a numerical expression for the tendency of two related variables to increase or decrease simultaneously. If the variations of each variable be represented by plus or minus signs, there will be n pairs of signs, which may be either like or unlike. Let c denote the number of pairs of signs, which predominate. Then the coefficient of correlation is $r = \pm\sqrt{\frac{(2c-n)}{n}}$. The coefficient is positive or negative, as the pairs with like or unlike signs predominate. The coefficient by this method is theoretically identical, when n is large, with that by the ordinary method when the variations between successive items, instead of departures from the mean, are employed.

Conclusion.—The statistical and other evidence thus far adduced in favor of a period between 2 and 2½ years is cumulative to a degree that is almost conclusive as to its existence. Moreover this evidence is mostly of a direct mathematical character that entirely precludes any personal bias entering into the results.

III. EVIDENCE FOR VARIABILITY IN THE LENGTH OF PERIODS

The question as to the existence of periods of systematically varying length is a fundamental one in this discussion and the evidence for the existence of such features in real periods will now be reviewed.

Variable stars.—The intervals between maximum brilliancy of some long-period variable stars are, it is well known, subject to marked variations in length. This variability is not of an accidental nature due to errors of observation or paucity of data, but must be regarded as systematic.

Statistical evidence for the systematic variability of the sun-spot period.—This period is well known to be of variable length, with an extreme range from 7 to 16 years, during the 310 years from 1610 to 1920. In this period of time we find by actual count that one-third of the intervals between successive maxima or successive minima are less than 10 years or greater than 12 years. I have applied various statistical criteria to Newcomb's [14]

[12] Clayton, H. H., "A lately discovered meteorological cycle." AM. MET'L JOUR., 1: 130, 1884.

[13] Douglas, A. E., "Climatic cycles and tree growth." Carnegie Institution of Washington, Pub. No. 289, p. 106, 1919.

[14] Newcomb, Simon, "On the period of the solar spots." Astro. Jour. 13:1. 1901.

residuals, i. e., the differences between his computed normal epochs and the observed epochs for a period of 300 years. The mean deviation of the residuals of the epochs of maxima is 1.27 years, and of the minima 1.00 years; while the mean variabilities are 1.49 and 1.26 years, respectively. If the sequence of the residuals were of a fortuitous character, the ratio, mean variability divided by the mean deviation, would be 1.41[15] instead of being actually 1.21. The latter ratio being so much less than the theoretical ratio for unrelated numbers indicates the existence of marked systematic characteristics in the residuals.

A further criterion is one based on the relative number of changes of sign in a series of residuals. The percentage of the whole number of such sign changes to the total number of residuals should be 50 for unrelated numbers. The percentage for the sun-spot residuals is 35.

The systematic character of the sun-spot intervals is shown by another criterion based on a comparison of results obtained by means of a smoothing formula. If the successive values, a, the interval from minimum to maximum, and b, the interval from maximum to minimum, be tabulated for the entire period from 1600 to 1900 and smoothed by the formula $\frac{a+2b+c}{4}$, there are obtained 17 well defined maxima and minima for the total 54 phase values. Assuming 8.5 as the average number of maxima or minima, it follows that the average interval between maxima or minima is 6.4 phase values. The corresponding mean interval for unrelated numbers smoothed by this formula is 4.7, a value determined empirically by the writer. The average phase value is 5.56 years or the average of a and b; hence the average interval is $5.56 \times 6.4 = 35.6$ years or the Brückner period, which is thus shown to exist in solar data.

Summarizing the statistical evidence cited above, we find strong evidence of the systematic variability of the length of the sun spot period. Accordingly, those who oppose the claim that the length of the sun spot period *varies systematically* must refute or otherwise adequately interpret the impersonal statistical evidence thereof just cited.

Empirical evidence of periodic variation in length of sun spot period.—In an earlier paper [16] I showed that this systematic variation in the length of the sun spot period was a periodic variation, in the sense that recurrences of long or short periods occur at intervals varying from 25 to 45 or 50 years, with an average interval of about 36 years, and supported this by curves showing similar secular variations in the spot numbers at maxima, and in the ratio $a:b$, or the ratio of the ascent from minimum to maximum to the descent from maximum to minimum. The close synchronism between the variations of these three solar elements is strong evidence as to the reality of the 36-year period. It was further shown that this 36-year secular variation in the length of the 11-year sun spot period which has persisted since 1600, is also well marked in the "probable maxima" of Fritz [17] since 1000 A. D. The latter are based partly on Chinese observations of spots visible to the naked eye, and partly on observations of the aurora in Europe.

Relative accuracy of old and modern sun spot data.—Since the value of the mean deviation of Newcomb's residuals is greater for the epochs of the maximum of spottedness than for the epochs of minimum, we must conclude that the latter recur with greater regularity or that their dates are more definite and more easily fixed. Furthermore, notwithstanding the much greater weight Wolfer has given to later observations of spottedness, the mean deviation of the residuals for the interval 1610 to 1750 is only slightly in excess of that for the interval 1750 to 1900. The proper interpretation to put upon this result is this: The causes which produce the considerable irregularities in the lengths of the periods are mostly systematic and due to some law of nature, whereas actual errors in determinations of the epochs are relatively small and have but a slight effect, so that the old and supposedly inaccurate observations are really quite as trustworthy for purposes of fixing the mean length of the sun spot period and its variations as the more exact and detailed modern data.

This statement applies also to the "probable maxima" of Fritz, the general accuracy of which, notwithstanding the apparent inadequacy of the data upon which they are based, is indicated by the persistency with which recurrences of long or short intervals between maxima, averaging 36 years apart, have appeared throughout the series. The writer has recently completed a statistical analysis of these early sun spot maxima, which in a remarkable manner confirms their general accuracy and their coordination with modern data.

Periodic variations in the length of the Brückner period.—Passing from solar to terrestrial data, it is to be noted that the Brückner period is also a variable period. Brückner fully recognized the variability in its length, as shown by his table of frequencies for various period lengths.[18]

Period lengths...years..	20	25	30	35	40	45	50
Frequencies...cases..	6	10	12	13	12	8	4

These values yield a mean length of about 35 years, with a probable error of ± 0.7 year. The extremes of 20 and 50 years may be regarded as due chiefly to the phase-shifting influence of shorter periods, but there can be no question that the length of the period has varied systematically between 25 and 45 years.

In the paper referred to above I showed that there is a close synchronism between the solar 36-year period and the Brückner period, the length of which I determined to be about 36 years. Short sun spot intervals, ranging from 7 to 10 years, invariably precede by an average of 10 years Brückner's epochs of cold winters. This close relationship is mutually confirmatory of both Brückner epochs from 1000 A. D. and the sun spot epochs of Wolfer from 1610, as well as the "probable maxima" of Fritz, from 1000 A. D. to 1600 A. D.

It was still further pointed out that the Brückner period and also the 36-year solar period show synchronous secular variations in length, ranging from 25 to 45 years, and evidence was given to show that this periodic recurrence of long and short intervals has occurred about every 300 years.

Variability of the 7-year period.—Another meteorological period of variable length is the period of about 7¼ years, which the writer has extensively investigated. A partial summary of this investigation, containing curves of temperature, pressure, and rainfall for the United States, was published in 1920.[19] The tempera-

[15] Ch. Goutereau, "Sur la variabilité de la température." Annuaire de la Société météorologique de France, 1906, 54; 122.

[16] Clough, H. W., "Synchronous variations in solar and terrestrial phenomena," Astro. Jour. 22: 59, 1905.

[17] Fritz, H., "Die Perioden solarer und terrestrischer Erscheinungen." Viertel. der Nat. Gesell. Zurich, 1893.

[18] Brückner, E., "Klimaschwankungen seit 1700," Geogr. Abhand. Band 4, Heft 277, Vienna, 1890.

[19] Clough, H. W., "An approximate 7-year period in terrestrial weather with solar correlation." MO. WEATHER REV., 48: 593-597, 1920.

ture curves from 1790 show that the intervals between maxima or minima have ranged between 5 and 10 years, and a smoothed plot of these period intervals shows close correlation with a curve of variations in the length of the 11-year sun spot period. Both curves show well marked secular variations with a period of 30 to 35 years, the terrestrial curve following somewhat the solar curve.

Conclusion.—The evidence, therefore, seems to indicate that variations in the periods of the sun and stars may occur, not of a purely accidental type due to uneliminated errors of observation or lack of adequate data, but definitely shown to be of a systematic and even of a periodic nature, and that similar variations occur in terrestrial periods, as the 36-year Brückner period and the 7¼-year period.

It would seem, therefore, in keeping with the evidence thus far adduced, that the attitude of one approaching the study of short-period variations in weather should at least be one of open-mindedness regarding the question of systematic or periodic variations in the length of such periods.

IV. METHODS OF INVESTIGATING PERIODICITIES

Among several important methods which investigators of weather sequences may employ, two will be contrasted and discussed somewhat at length: (1) An empirical method based largely upon careful examination of curves drawn free-hand through plotted data or derived by means of smoothing formulae; (2) a mathematical analysis, illustrated by the various applications of the harmonic analysis, including the Schuster periodogram and a new graphical periodicity tabulation to be described below. These methods are not in any sense exclusive of each other; rather, the second method may be said to be supplementary to the first, with the object of obtaining a possible quantitative evaluation of the results derived thereby.

The following extract from a paper by Bigelow [20] gives an impartial statement as to the relative merits of the two methods:

> The problem is so exceedingly difficult that we may fairly be permitted to employ such means of discussion as are obviously suitable to avoid an inevitable failure in reaching a valuable result. The problems of solar and terrestrial synchronism can be discussed by two general methods—a strictly rigid mathematical method, and a statistical method, combined with an interpretation guided by graphic traces. The former is preferred by some as applying definite principles and allowing no chance for accomodation by a biased judgment; the latter is preferred by many as the only method for a first approximation to a clear understanding of relations too complicated to be unraveled by any mathematical method now in existence. Some criticize the former method as allowing no room for the practical judgment, and others criticise the latter method as allowing too much room for the judgment, especially on the part of those who seek a special result. The truth seems to be that the former method is allowable for the adjustment of the constants and terms of an equation, wherein the physical processes are already approximately understood. The latter method is necessary and allowable in those preliminary researches which seek to discover what the law is rather than in the refinement of it. The first method always leads to zero results in dealing with solar and terrestrial phenomena; the latter offers some hope of success in the present state of the development of the science.
>
> We mean to include under the former method of rigid analysis (1) the usual application of the theory of least squares for the detection of an unknown period; (2) Professor Schuster's "Harmonic analysis and periodogram for the detection of hidden periodicities," Terr. Mag. 1898; (3) the Fourier series and sequence in various forms of the harmonic analysis; (4) Professor Newcomb's "Criterion for fluctuations without any discernible period," Tran. Am. Phil. Soc. vol. XXI, part V, 1908.

First method.—As an example of the first method, the determination of the epochs of sun spot maxima and minima may be cited. Schwabe observed the sun for little more than three complete periods and on this basis announced his discovery, which was gradually accepted by astronomers. In this case the range between maxima and minima is so great that a simple inspection of the curves representing the observations is convincing. Later, Wolf and his successor Wolfer compiled all available observations as far back as 1605, and showed the average length to be 11⅛ years, with variations ranging from 7 to 16 years.

However, the long interval from the maximum of 1788 to the maximum of 1805 was regarded by some as a double period with an intermediate primary maximum. Young states (The Sun, p. 149): "Some astronomers contend that there ought to be another maximum included about 1795. Observations about this time are few in number and not very satisfactory." A controversy over this matter lasted many years until Wolf definitely settled the question and showed that magnetic and auroral data are confirmatory of the long period.

Wolf [21] determined empirically the sun spot epochs by means of his smoothed numbers consisting of consecutive 12-month means still further smoothed by taking means of two consecutive 12-month means centered on the first of the contiguous months, giving a result as of the 15th of the first month. Some maxima in spite of the smoothing showed double crests and in such cases the epoch was assigned to a point intermediate between the crests. The method is entirely empirical and another might assign slightly different epochs from an inspection of the curves. The fact that variations in the diurnal range of magnetic declination show high correlation with the sun spot variations, as do also auroral data, may be said to confirm the accuracy of the empirically determined sun spot epochs.

The determination of Brückner period is a further illustration of the first method of investigation. Brückner employed 5-year means as units and deduced from simple inspection of the data, in both tabular and graphic form, the dates of maximum and minimum values of the period which bears his name.

Second method.—When analytical methods which involve the assumption of a uniform period are employed, an obstacle immediately presents itself. The variability in the length of the period rapidly reduces the amplitude derived from a periodicity tabulation of more than a few rows. The amplitude of the sun spot period, for example, is reduced nearly 50 per cent if the yearly values are tabulated by 11-year recurrences from 1750, owing to the varying length of the period. For this and other reasons the many investigations in which the Fourier analysis has been employed have failed to give certain evidence of any period of uniform length in weather data.

A noteworthy example of this method is Brunt's [22] discussion of the monthly residuals of temperature at Greenwich, 1841-1905, which led to negative results. He states:

> The investigation of the continuity of the different periods considered above points to the conclusion that there exists no stable periodicity except the annual one. If this conclusion is justified, it is clear that the periodogram analysis is not in itself sufficient to deal with temperature variations.

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[20] Bigelow, F. H., "Studies on the general circulation of the earth's atmosphere." Am. Jour. Sci. 29: 281, 1910.

[21] Wolfer's revised monthly relative numbers, actual and smoothed, with a discussion of the method employed in deriving the epochs of maxima and minima will be found in Mo. WEATHER REV. 30:171, 1902.

[22] Brunt, D., "A periodogram analysis of the Greenwich temperature records.' Quart. Jour. Roy. Meteor. Soc. 45:335, 1919.

Perhaps the most notable example of the periodogram method is afforded by Beveridge's [23] investigation of weather and harvest cycles. He analyzes wheat prices in western Europe based on data for 300 years and finds a number of apparent periods with amplitudes which he regards as sufficiently large to be significant. Efforts to support the reality of these periods by reference to supposed meteorological parallels are not very successful. If the systematic variability in the length of meteorological periods is admitted, this method has limitations upon its use, and results so secured must be interpreted in the light of those limitations, particularly for the shorter periods, where 20 to 30 or more recurrences are involved.

Probably the most noteworthy feature of his periodogram is the relatively large amplitude appearing at 35.5 years. This length of period is approximately that of the Brückner period, and the phases of maximum wheat prices agree closely with Brückner's cold, wet epochs. There being only eight recurrences, a fairly large amplitude survives despite the variation in the length of the period. Beveridge's results may be said to be definitely confirmatory of the reality and importance of the Brückner period.

It is highly probable, therefore, that if there exists any real period of uniform length it would have been discovered by now, since numerous investigators have employed methods of mathematical analysis which would inevitably result in the emergence of such a period if it existed. In view of the further fact that the sequence of values in meteorological data, particularly for temperature, is systematic to a high degree, the inference is that the recurrent features thus indicated are variable to an extent sufficient to limit the usefulness of refined analytical methods and to make difficult and uncertain the interpretation of results so attained.

Discussion of Wallén's[24] methods and results.—The investigations of Wallén will now be given somewhat detailed consideration because they constitute a good example of the use of the empirical method of analysis. He has for a number of years issued annual forecasts of the level of the Swedish lakes Malar and Wener, based upon the short period of 2½ years and an 11-year period. He employed the rainfall record at Upsala from 1740, the temperature at Stockholm from 1757, and the stages of Lake Malar from 1774, to determine the normal periods and amplitudes which he employs in making his forecasts.

The fundamental principle of his method consists of the methodical elimination of the different periods, proceeding from the shortest, or annual, to the longest by the formation of mean values of continuous groups having a number of terms equal to that of the period to be eliminated. Thus the annual period is eliminated by running 12-month means, still further smoothed by consecutive means of 5 terms. The 2½-year period is then eliminated by combining these means into consecutive means of say, 36 terms. The differences between these two series yield the 2½-year period with the annual and secular variations eliminated. These operations do not alter the length or the phase of the period; but the amplitude suffers diminution, for which, however, it is easy to make a correction, as will be shown.

While the present writer fully approves of the method used by Wallén, it seems necessary to digress briefly at this point in order to present and analyze the conclusions reached by him and, if possible, to harmonize his findings concerning the periodicity of about 2½ years with the findings of the present investigation, as set forth below. From the curves obtained by the methods outlined above, Wallén derived empirically, by simple inspection, the epochs of maxima and minima of the 2.5-year period, thereby obtaining a mean of 30 months for Lake Malar, 24 months for the rainfall at Upsala, and 26 months for the temperature at Stockholm. Assuming the amplitude of the annual period for each element as 1.0, he derived a value of 0.94 for the amplitude of the 30-month period in the variations of Lake Malar, and 0.44 for the amplitude of the 30-month period in the rainfall at Upsala. Wallén's phase-intervals for Lake Malar when arranged in a frequency tabulation show a mode at 28 months. The mode for the rainfall phase-intervals is about 23 months, while the temperature at Stockholm yields a mode at 24.5 months.

These differences in Wallén's results for three meteorological elements at practically the same locality are due to the fact that in the rainfall data certain minor maxima and minima occur which do not have a corresponding feature in the variations of the lake level, owing to the storage effect of the lake. The phases of the 30-month variation in the lake level occur normally about 2 months after corresponding phases in the rainfall. Thus Wallén derives a value for the period in the lake levels somewhat greater than that for the rainfall. Owing to the fact that the amplitude of the annual variation in the lake level is but little in excess of that of the 30-month period, while in the case of the rainfall it is more than double, it is reasonable to infer that the variations in the lake level represent more nearly the actual short-period variations, since the annual variation, being relatively much smaller, can be more perfectly eliminated than in the case of rainfall. Furthermore, it is irrational to suppose that elements as closely related as rainfall and lake levels in the same locality should not theoretically have closely corresponding maxima amd minima with precisely the same average length of period. Wallén has also investigated the levels of Lake Wener from 1807. He finds that maxima and minima in this lake occur slightly later than corresponding phases for Lake Malar, and that certain fluctuations in the latter are not clearly obvious in the larger lake, and he accordingly derives a mean period of 33 months for this lake. These differences are due solely to the greater storage capacity of the larger lake, resulting in occasional fusion of two consecutive maxima or minima of the smaller lake. It will be shown below why my epochs for the level of Lake Malar differ in some respects from those of Wallén.

The 12-month smoothing process.—Resuming again the consideration of empirical methods with special reference to the elimination of possible other short-length periods, the primary object of any smoothing process is to eliminate minor variations in order to present more clearly important major variations. The particular formula to be used depends upon the precise object in view.

The objects to be attained by the running 12-month mean are twofold: (1) The elimination of the annual variation. This procedure is, as a rule, effective for this purpose; but in the case of rainfall, at times when the annual variation is not normal, this elimination is not perfectly accomplished and uneliminated effects may cause a slight shifting in the phase of the longer period at such times. Such phase shiftings may, however, be regarded as deviations of an accidental nature and could

[23] Beveridge, Sir William. "Wheat prices and rainfall in western Europe," Jour. Roy. Statis. Soc. 85:412-478, 1922.

[24] Wallén, A., "Fleråriga Variationer hos Vattenståndet i Mälaren, Nederbörden i Upsala och Luft-temperaturen i Stockholm." Meddelanden från Hydrografiska byrån, 4. Stockholm, 1913.

not amount to more than a quarter of a year at the most. (2) Elimination of possible periods of under and slightly over one year in length. According to a formula due to Schreiber,[25] we can determine how much the amplitude of a periodic function is weakened through the formation of running means with a certain number of terms.

Let the function be

$$y = a \sin\left(\frac{180°}{l} + b\right)$$

in which a = the amplitude
l = the length of the period.

If running means of n terms be found, there is obtained the new periodic function

$$\eta = \mu\, a \sin\left(\frac{180°}{l} + b\right)$$

where μ is a factor that expresses how much the amplitude is diminished. μ has the following form:

$$\mu = \frac{\sin n\left(\frac{180°}{l}\right)}{n \sin\left(\frac{180°}{l}\right)}$$

in which n and l are in the same units.

I have computed the value of this factor μ for various lengths of periods up to 36 months where n=12. The results are shown in the following table, in which l represents the length of the period and μ the percentage of the true amplitude.

l	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	20	24	30	36
μ	0	0	0	−13	0	−15	−22	−21	−16	−8	0	+8	+16	+24	+30	+41	+51	+64	+76	+83

It appears that periods commensurate with 12 months, as, 2, 3, 4, and 6 months are entirely eliminated, while incommensurate periods, as 7 to 11 months, appear with reversed phase (shown by the minus sign) and greatly reduced amplitude, amounting to only about one-fifth the original magnitude for the maximum possible effect at 8 or 9 months. One may reasonably infer from this—and a study of numerous curves is confirmatory—that the effect of periods shorter than one year is practically negligible and confined to a slight shifting of phase which may be regarded as of an accidental nature. In actual practice, the further smoothing of the 12-month means by consecutive means of five terms effectually eliminates all possible periods under 1 year.

As to the existence of possible periods longer than 1 year the table shows that the amplitude of periods up to 16 months is reduced to less than one-third of the original amount, while with an additional five-term smoothing the reduction is to 26 per cent or less, so that these may be practically disregarded in the 12-month means. As to the possible existence of periods between 16 and 24 months, the evidence of the curves is emphatically in the negative. The existence of such periods would be shown by the frequent occurrence of minor or supplementary crests or hollows, and such features are relatively infrequent in the temperature and pressure curves.

The practical absence of such supplemental phases in the 12-month means indicates that the amplitude of possible periods between 16 and 24 months is very much smaller than that of the 28-month period and that their only effect would be an apparent shifting of the phase, which may be regarded as of the nature of an accidental deviation. These several accidental deviations of the phase due to abnormal annual variation, and possible short periods should as a rule neutralize each other, and the best possible evidence that they generally do is furnished by a frequency tabulation of the phase intervals of some of the curves examined. There is no *a priori* reason why a 20-month interval, for example, should not appear as the chief mode in such a tabulation, since theoretically it should appear in the curve with 51 per cent of its original amplitude. As a matter of fact the most frequent interval for the variations of Lake Malar, for example with annual and secular variations eliminated, is about 28 months and the distribution is unimodal and nearly symmetrical, showing that the deviations are of an accidental nature.

A curve of running 12-month means exhibits numerous small fluctuations superposed upon the long variation of two or more years and these may be smoothed out, as did Wallén, by forming consecutive means of five terms. By employing only two of the these 12-month means, 6 months apart, nearly the same result is effected, with, however, a corresponding reduction in amplitude, which is shared to an even greater degree by the possible short periods. For a 30-month period the 12-month smoothing effects a reduction in amplitude to 76 per cent, according to the above table giving the values of μ. A selection of two of the 12-month means, 6 months apart, effects a still further reduction, amounting to about 90 per cent. Hence the original amplitude of the period is reduced to 0.76×0.90 or 68 per cent by this process. For a 24-month period the resulting amplitude is about 60 per cent.

Methods employed in the present investigation.—In this investigation I have employed the ordinary calendar-year mean centered on July 1 and the 12-month mean centered on January 1. In deriving the latter mean the two half-yearly means beginning January 1 and July 1 of each year were computed, their mean being checked with the published annual mean, and finally the 12-month mean centered on January 1 was determined by averaging the second half of each year with the first half of the next. The computation of this mean involves only slight labor compared with that of 12-month consecutive means, which is prohibitive when the research involves the amount of data necessary to results of adequate generality. The use of these two means six months apart has given good satisfaction, since the annual period is eliminated in the only manner entirely satisfactory, and the months with large departures are grouped together in the January 1 mean. When it is recognized that two points suffice to completely fix a sine wave of given wave length if the coordinates are given with reference to the axis of the wave, it is clear that irregular wave forms may still be fairly well defined by from four to six points per wave. Thus values of data at intervals of 6 months are found generally sufficient to fairly well determine, within the limits of accidental deviations, the epochs of maxima and minima of a variable 2½-year wave within about a quarter of a year.

The fact that the January 1 mean embraces contiguous winter months of large departures, taken in connection with the average length of the period, 2 to 2½ years, renders this mean of relatively greater importance than the July 1 mean. This is also shown by the tendency for maxima and minima to occur more frequently in the

[25] Schreiber, P., "Vier Abhandlungen über Periodizität des Niederschlages." Abhand. der Kgl. sächs. meteorl. Institutes. Heft 1 Leipzig, 1896.

period centered on January 1 than in the 12-month mean centered on July 1, in which the winter months stand nearly a whole year apart and will differ in character widely at times.

A nearly ideal preparation of the data would consist of plotting overlapping 12-month means of the ratio, *monthly departures divided by the normal monthly mean deviations of the data*, a treatment which should produce very homogeneous values. However, a trial of this method has not shown sufficiently material differences to justify, especially in a preliminary study like the present one, the great additional labor the use of that method entails.

While, as a rule, the maxima and minima which appear in the curves secured by the methods described are not as well defined as those of the sun-spot curve, nevertheless, at certain stations and for certain elements, the recurrence of these phases fairly approximates, in respect of uniformity of phase interval and amplitude, these features of the solar curve. This is the case, for example, with the pressure at Portland, Oreg., when the secular trend is eliminated and the short period is completely segregated (cf. fig. 1). In some cases when the secular trend is not eliminated the existence of a maximum or minimum phase is indicated only by an inflexion in a continuous ascent or descent of the curve, due to a longer period. The amplitude of the short period is, however, as a rule sufficiently large in comparison with that of the variations of a longer period, as the 7-year period, so that the determination of the phases is quite unaffected by the existence of the longer period. Ordinarily when only the epochs of maxima and minima are desired, allowance can easily be made, by simple inspection, for the secular trend. In doubtful cases one must resort to curves for near-by stations for confirmation, hence it is necessary to have for close comparison curves for a number of stations for any particular region. In this manner instrumental errors, discrepancies due to changes of exposure or location, and the small differences normally occurring between localities more or less widely separated can also be eliminated by a visual comparison of the curves.

The method therefore is essentially that employed by Wolfer in determining the solar epochs, or by astronomers in determining the epochs of maxima and minima of variable stars. The evidence for the reality of the epochs differs only in the degree of certainty from that available for the determination of the solar epochs. The phase-intervals, while being far from uniform, only occasionally show any abrupt changes from short to long, and the amplitude is sufficient to render infrequent such uncertainty as for a long time prevailed regarding the sun-spot interval from 1788 to 1805.

V. THE 28-MONTH PERIOD IN METEOROLOGICAL ELEMENTS

Data employed in the investigation.—In the present investigation I have examined the temperature data in the United States for practically all the stations with long records. The 12-month means centered on January 1 and July 1 were computed and curves drawn for each station. In the years previous to 1830 all available records were employed. Pressure data since 1870 have also been examined for various regions, including the upper Mississippi Valley, the Canadian northwest, Oregon, Washington, British Columbia, and Alaska. In Europe data are available from 1725 for temperature and from 1740 for pressure. From three to six or eight contemporaneous records covering the entire period were plotted for study. In Europe the temperature data employed have been confined to Holland, northern Germany, Sweden, and northern Russia. For reasons which have previously been referred to, northern Europe exhibits the short period more satisfactorily than regions farther south. Similarly in the United States, the middle and upper Mississippi Valley, upper Ohio Valley, and the lake region yield curves with fluctuations more nearly representative of the short period than places farther south or west. The plateau region and Pacific States are characterized by fluctuations with phases differing markedly from those east of the Rocky Mountains.

For pressure, records in Holland, France, Italy, and in later years, Spain have been employed. It is found that southwestern France and western Spain on the whole show the short period most satisfactorily. This may be associated with the fact that the Atlantic high-pressure area projects onto the Continent of Europe over these regions. Other regions studied in detail as regards pressure variations are northern Egypt, India, Australia, Iceland, and Greenland.

The object of the investigation has been to establish by inspection of plotted curves the epochs of maxima and minima for the United States and Europe and to deduce from these epochs the average lag or phase interval between distant regions, and the underlying secular variations in the length of the period, which requires examination of the earliest records available.

For illustrating in more detail the conditions since 1870 there are given in Figure 1 curves representing variations in temperature, pressure, rainfall and other meteorological data at selected stations. The data plotted are 12-month means centered on January 1 and July 1, or two values per year. Several stations in the same region are shown to illustrate the degree of correlation likely to exist between two stations in the regions regarded as best showing the short period. The composite curve of temperature is an average of six stations, viz, Minnedosa, Winnipeg, Bismarck, Moorhead, Duluth, and St. Paul.

Figure 2 represents fluctuations of temperature and pressure previous to 1870. The European temperature curve is a composite of two to five stations, which, with the years of data employed, are as follows: Utrecht, 1729–1739; Leyden, 1740–1753; Berlin, 1730–1751; Åbo, 1750–1761; Lund, 1753–1773; St. Petersburg, 1752–1763, 1768–1800, 1806–1908; Stockholm, 1756–1802; Riga, 1796–1814; Wöro, 1800–1829; Torneo, 1801–1829; Archangel, 1814–1830, 1833–1908; Ustssyssolsk, 1818–1868; Jockmock, 1862–1918. The earlier years include data from Holland and Germany, but from 1750 only data from Sweden and northern Russia were employed. During the whole period of time three to eight contemporaneous curves were available and a section of a curve at any particular station which was markedly at variance with the curves of surrounding stations was rejected in averaging the station departures for the composite curve. Thus the data at St. Petersburg for 1765–1767 were rejected. All other data from the stations above named were used in deriving the composite curve.

For the European pressure curve, data at the following stations were employed, viz: Amsterdam, 1740–1761; Paris, 1758–1853; San Fernando, 1850–1919. This curve therefore represents mainly data from only a single station. As in the case of temperature, several contemporaneous curves were available for study and comparison and these stations for the period of time stated were selected as being most representative of the variations

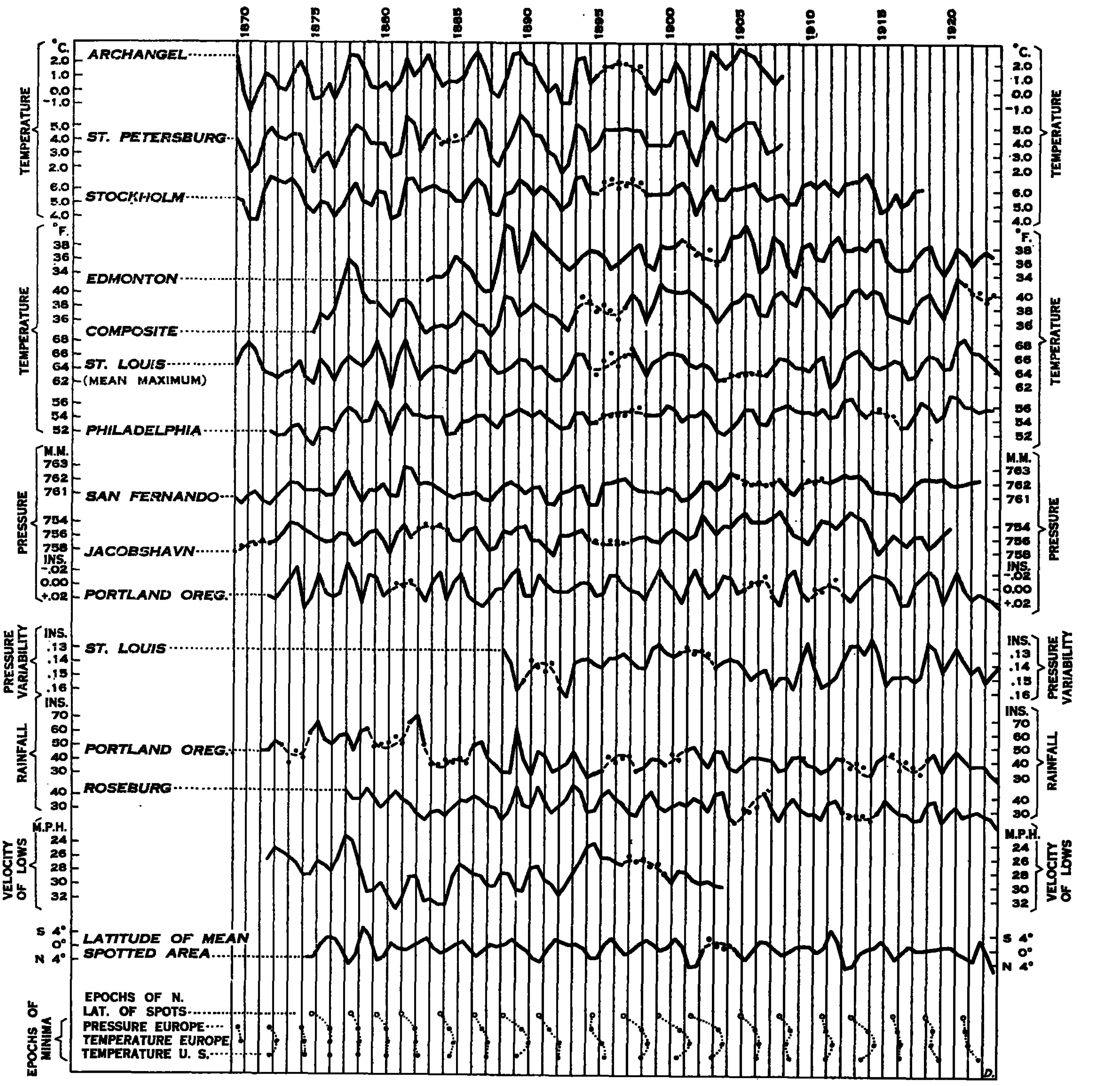

FIG. 1.—Curves representing the 28-month variations, since 1870, in pressure, temperature, rainfall, and pressure variability for selected stations in Europe and the United States, the velocity of areas of low pressure in the United States, and the mean latitude of solar spots. Smooth curves are drawn where minor fluctuations occur. The location of associated phases of temperature, pressure, and latitude of spots is indicated by dots and circles with connecting dotted lines

over southwestern Europe. It is well known that the correlation between the variations in pressure at stations more or less widely separated is considerably greater than the correlation between their variations in temperature, so that the pressure data, if homogenous, from a single station, may yield results entirely representative of a comparatively large region. The pressure data at Paris give internal evidence as well as evidence derived from comparison with other stations as to their fully representative character. Observations at San Fernando were available from 1850 and were regarded, in view of their homogenous character and agreement with neighboring stations, as representative of pressure variations in western Spain. Their correlation with temperature variations in northern Europe is higher than that derived from the pressure data at Paris. Being in 12° lower latitude than Paris, the range of fluctuation is correspondingly less.

For the United States the temperature curve is a composite of the data at the following stations: New Haven, 1780–1828; Salem, 1786–1828; Winslow, Nova Scotia, 1794–1811; Cincinnati (College Hill), 1814–1848; Cincinnati, 1835–1875; Marietta, 1826–1875; St. Louis, 1833–1875. The earlier data are thus drawn mainly from New England. From 1828 the upper Ohio Valley and middle Mississippi Valley contributed to the composite curve, which is thus based on from two to four stations. The range of variation in the Middle West and in Nova Scotia is greater than in southern New England.

Epochs of maxima and minima for the data studied.—Employing the methods previously described, the epochs of minimum temperature and minimum pressure of the short period have been derived for Europe since 1730, and the epochs of both maximum and minimum temperature for the United States since 1780. These epochs are given in Table 1. They have been derived by comparison of all the curves drawn, having due regard at the same time to the secular trend. For this reason the epochs may at times differ slightly from those apparently indicated by the composite curves in Figure 2. The object of the investigation, namely, the determination of the mean length of the period and its secular variations, has been attained in the case of European data in a satisfactory manner by consideration of minima only. Table 1 gives also for each epoch the phase interval from the preceding epoch.

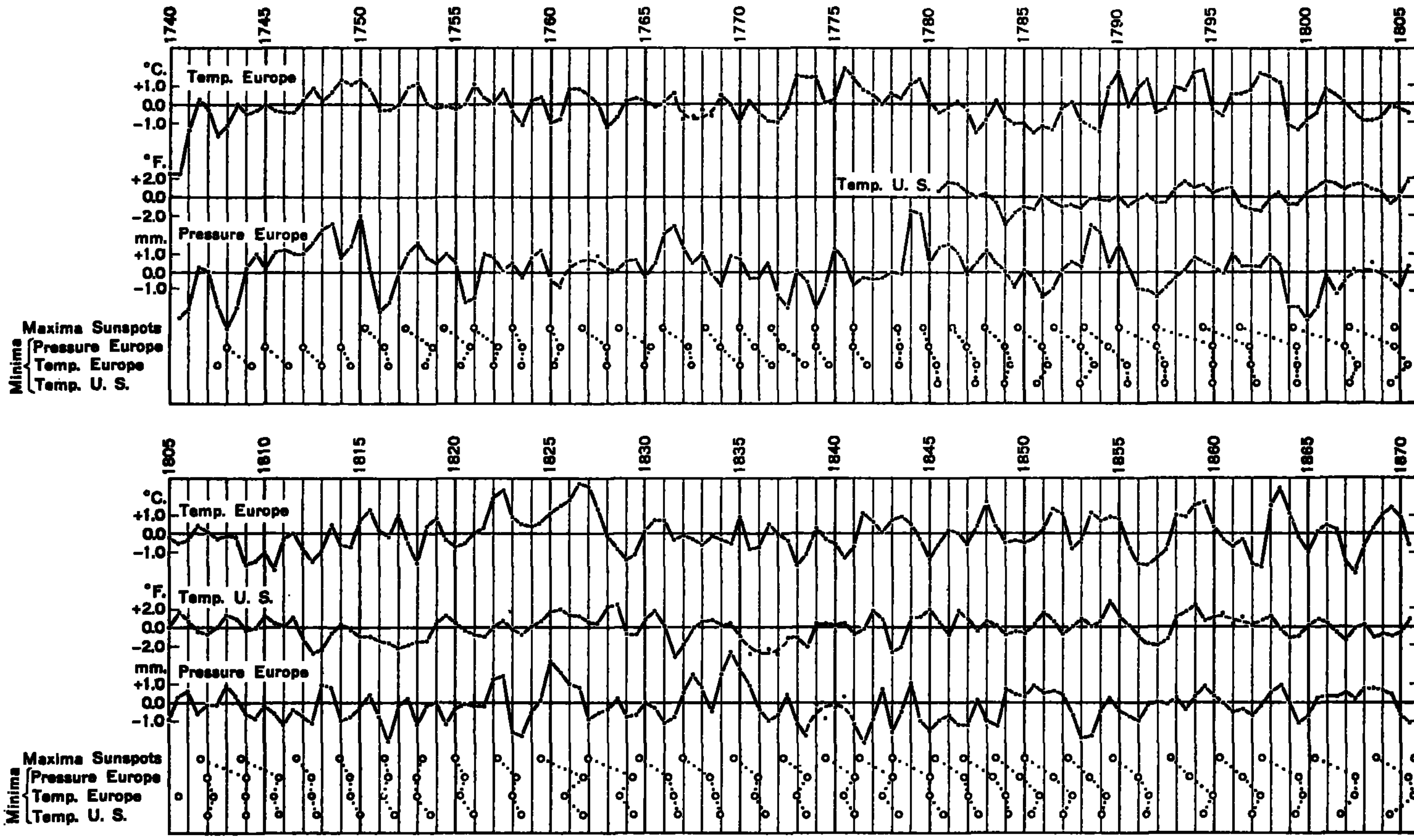

FIG. 2.—Curves (composite) representing the [illegible]-month variations in temperature and pressure in Europe and the United States from 1740 to 1870. The epochs of sun spot maxima and associated phases of temperature and pressure are indicated by circles with connecting dotted lines

The minima of the short period for pressure and temperature in Europe and for temperature in the United States are indicated at the bottom of Figures 1 and 2 by symbols plotted on the dates given in Table 1. These minima are regarded as being associated in a relation of approximate synchronism, and hence as being causally related, in the manner indicated by dotted lines joining the appropriate symbols.

The average interval from maxima to minima is nearly the same as from minima to maxima, and the wave form may therefore be regarded as practically symmetrical.

TABLE 1.—*Meteorological and solar epochs*

Temperature in N. Europe		Temperature in the United States				Pressure in southwestern Europe, minimum	Solar spots, maximum
Minimum	Interval	Minimum	Interval	Maximum	Interval		
1729.0							
1731.0	2.00						
1732.7	1.75						
1734.5	1.75						
1736.0	1.50						
1738.0	2.00						
1740.5	2.50						
1742.5	2.00						
1744.2	1.75					1743.0	
1746.2	2.00					1745.0	
1748.0	1.75					1747.0	
1749.5	1.50					1749.0	
1751.5	2.00					1751.2	1750.2
1753.5	2.00					1753.7	1752.5
1755.2	1.75					1755.7	1754.5
1757.0	1.75					1757.2	1756.0
1758.5	1.50					1758.5	1758.0
1760.2	1.75					1760.5	1760.0
1763.0	2.75					1763.0	1761.7
1765.0	2.00					1765.2	1763.7
1767.5	2.50					1767.2	1766.0
1770.0	2.50					1769.0	1768.2
1771.7	1.75					1770.7	1770.0
1773.5	1.75					1772.2	1771.7
1775.0	1.50					1774.0	1774.0
1776.7	1.75					1776.0	1776.0
1778.7	2.00					1778.5	1778.2
1780.5	1.75	1780.5		1781.2		1780.0	1779.7
1782.5	2.00	1782.5	2.00	1783.2	2.00	1782.0	1781.2
1784.2	1.75	1784.0	1.50	1784.7	1.50	1784.0	1783.0
1786.2	2.00	1785.7	1.75	1786.5	1.75	1786.0	1784.7
1788.7	2.50	1788.0	2.25	1789.2	2.75	1788.0	1786.7
1790.5	1.75	1790.5	2.50	1791.5	2.25	1789.5	1788.2
1792.5	2.00	1792.5	2.00	1793.7	2.25	1792.0	1790.0
1795.0	2.50	1795.0	2.50	1796.0	2.25	1795.0	1792.2
1797.0	2.00	1797.2	2.25	1798.5	2.50	1797.0	1794.5
1799.5	2.50	1799.5	2.25	1801.0	2.50	1799.5	1796.5
1802.7	3.25	1802.0	2.50	1803.0	2.00	1802.2	1799.2
1805.5	2.75	1804.5	2.50	1805.5	2.50	1804.7	1802.2
1807.2	1.75	1807.0	2.50	1808.0	2.50	1807.0	1804.7
1809.0	1.75	1809.0	2.00	1810.0	2.00	1809.0	1806.7
1810.5	1.50	1810.7	1.75	1811.5	1.50	1810.7	1808.7
1812.5	2.00	1812.7	2.00	1814.0	2.50	1812.5	1811.7
1814.5	2.00	1815.0	2.25	1816.0	2.00	1814.5	1814.0
1816.2	1.75	1817.0	2.00	1817.7	1.75	1816.5	1816.2
1818.0	1.75	1818.5	1.50	1819.5	1.75	1818.0	1818.2
1820.2	2.25	1821.0	2.50	1822.5	3.00	1820.5	1820.0
1823.0	2.75	1823.5	2.50	1825.2	2.75	1823.2	1822.2
1825.7	2.75	1826.7	3.25	1828.0	2.75	1826.7	1824.5
1828.7	3.00	1829.2	2.50	1830.5	2.50	1829.2	1827.0
1831.5	2.75	1831.7	2.50	1833.0	2.50	1831.2	1829.7
1833.7	2.25	1834.0	2.25	1835.0	2.00	1833.5	1832.0
1835.7	2.00	1836.2	2.25	1837.5	2.50	1836.0	1834.7
1838.0	2.25	1838.5	2.25	1839.5	2.00	1838.5	1837.2
1840.5	2.50	1841.0	2.50	1842.0	2.50	1841.0	1839.5
1842.5	2.00	1843.2	2.25	1844.2	2.25	1843.0	1841.2
1845.0	2.50	1845.7	2.50	1846.5	2.25	1845.0	1843.0
1847.0	2.00	1847.5	1.75	1848.2	1.75	1846.7	1845.2
1849.0	2.00	1849.0	1.50	1849.5	1.25	1848.2	1846.7
1850.7	1.75	1850.5	1.50	1851.0	1.50	1849.7	1848.5
1852.5	1.75	1852.0	1.50	1853.0	2.00	1851.5	1850.0
1854.2	1.75	1854.0	2.00	1855.0	2.00	1853.5	1852.2
1856.5	2.25	1856.5	2.50	1858.2	3.25	1856.0	1854.7
1860.0	3.50	1859.5	3.00	1860.5	2.25	1858.7	1857.7
1862.5	2.50	1862.0	2.50	1863.0	2.50	1861.5	1860.2
1864.7	2.25	1864.2	2.25	1865.5	2.50	1864.5	1862.5
1867.5	2.75	1866.7	2.50	1868.0	2.50	1867.5	1865.2
1870.7	3.25	1869.5	2.75	1871.0	3.00	1870.5	1868.7
1873.2	2.50	1872.7	3.25	1874.2	3.25	1872.7	1870.7
1875.2	2.00	1875.2	2.50	1876.0	1.75	1875.0	1872.7
1877.0	1.75	1877.0	1.75	1878.0	2.00	1877.0	1874.7
1879.0	2.00	1879.0	2.00	1880.0	2.00	1879.0	1877.2
1881.0	2.00	1881.0	2.00	1882.0	2.00	1881.0	1880.0
1883.0	2.00	1883.2	2.25	1884.5	2.50	1883.0	1882.0
1885.7	2.75	1885.5	2.25	1886.7	2.25	1885.5	1884.0
1888.2	2.50	1888.0	2.50	1889.5	2.75	1887.7	1885.7
1891.0	2.75	1890.2	2.25	1891.5	2.00	1890.5	1888.2
1893.2	2.25	1893.2	3.00	1894.5	3.00	1892.5	1891.5
1896.2	3.00	1896.5	3.25	1898.0	3.50	1895.5	1893.7
1899.5	3.25	1899.0	2.50	1900.2	2.25	1899.0	1896.0
1902.5	3.00	1901.5	2.50	1902.5	2.25	1901.7	1898.2
1904.7	2.25	1904.0	2.50	1906.0	3.50	1904.5	1900.7
1907.2	2.50	1907.2	3.25	1908.5	2.50	1907.0	1903.7
1909.2	2.00	1909.5	2.25	1910.5	2.00	1909.5	1906.7
1912.5	3.25	1912.0	2.50	1913.5	3.00	1912.0	1909.2
1915.5	3.00	1914.5	2.50	1916.0	3.00	1915.0	1912.5
1917.2	1.75	1917.2	2.75	1919.0	2.50	1917.0	1915.5
1919.5	2.25	1920.0	2.75	1921.2	2.25	1919.2	1918.0
1922.2	2.75	1922.5	2.50	1924.0	2.75	1921.2	1920.2
						1924.0	

NOTE.—The tenths .2 and .7 in the dates stand for .25 and .75 respectively.

The mean length of the period and analysis of its variations.—The result of the United States series from 1780 to 1920 yields an average interval of 2.33 years between like phases, with a probable error, derived from the mean deviation, of ±.038 year. The standard deviation of the 61 intervals between successive minima is 0.43 year, the mean deviation 0.33, and the mean variability, or the mean change between successive values is 0.32 year. Analyzing these measures of dispersion and of the order of succession we notice that the standard deviation is 1.29 times the mean deviation. Since this ratio agrees very nearly with Cornu's ratio, 1.25, we may conclude that the distribution is nearly according to the Gaussian curve. The mean variability (0.32) divided by the mean deviation (0.33) is 0.97, a ratio so much smaller than Goutereau's ratio 1.41, that we must conclude that the order of succession of the numbers can not be fortuitous or unrelated, but rather is determined by some physical cause or influence not yet known.

Referring to Figure 3, it is clear that there is a pronounced secular trend in the data which fully accounts for the smallness of this ratio. If residuals from the straight line of best fit to the data as a whole be computed, the mean deviation, or the mean of these residuals, would be much smaller than 0.33, the value obtained above, and the ratio, mean variability divided by mean deviation, would approach 1.41.

The criterion previously referred to in connection with the sun spot residuals yields a particularly impressive result when applied to the residuals of the European period-lengths. The probable percentage of the number of sign changes to the total number of residuals in a series with a fortuitous order of succession is 50. The percentage for the residuals of the European data is 30.

In Europe the results from 1730 to 1920 are, mean 2.20 years, standard deviation 0.47, mean deviation 0.39, and mean variability 0.38. The lower average for Europe is due to the inclusion of 50 years of data previous to the beginning of the United States record, during which time the mean length of the period was about 2.00 years.

Frequency of the phase intervals and differences.—The following frequency tabulation of the intervals between epochs in Table 1 shows a mode of two years for Europe and 2.50 years for the United States.

Phase intervals		
Years	Frequencies	
	Europe	United States
1.50	5	9
1.75	22	9
2.00	23	21
2.25	8	23
2.50	12	38
2.75	8	7
3.00	4	7
3.25	4	6
3.50	1	2
Total	87	122

Differences, Europe—United States	
Years	Frequencies
−1.00	1
−0.75	4
−0.50	9
−0.25	8
0.00	16
+0.25	6
+0.50	8
+0.75	5
+1.00	3
+1.25	1

A frequency tabulation of the differences between the European and the United States epochs is also shown above. The mean difference is +0.05 year, which means that the epochs of maxima and minima are, in the long run, practically coincident in the two continents. The

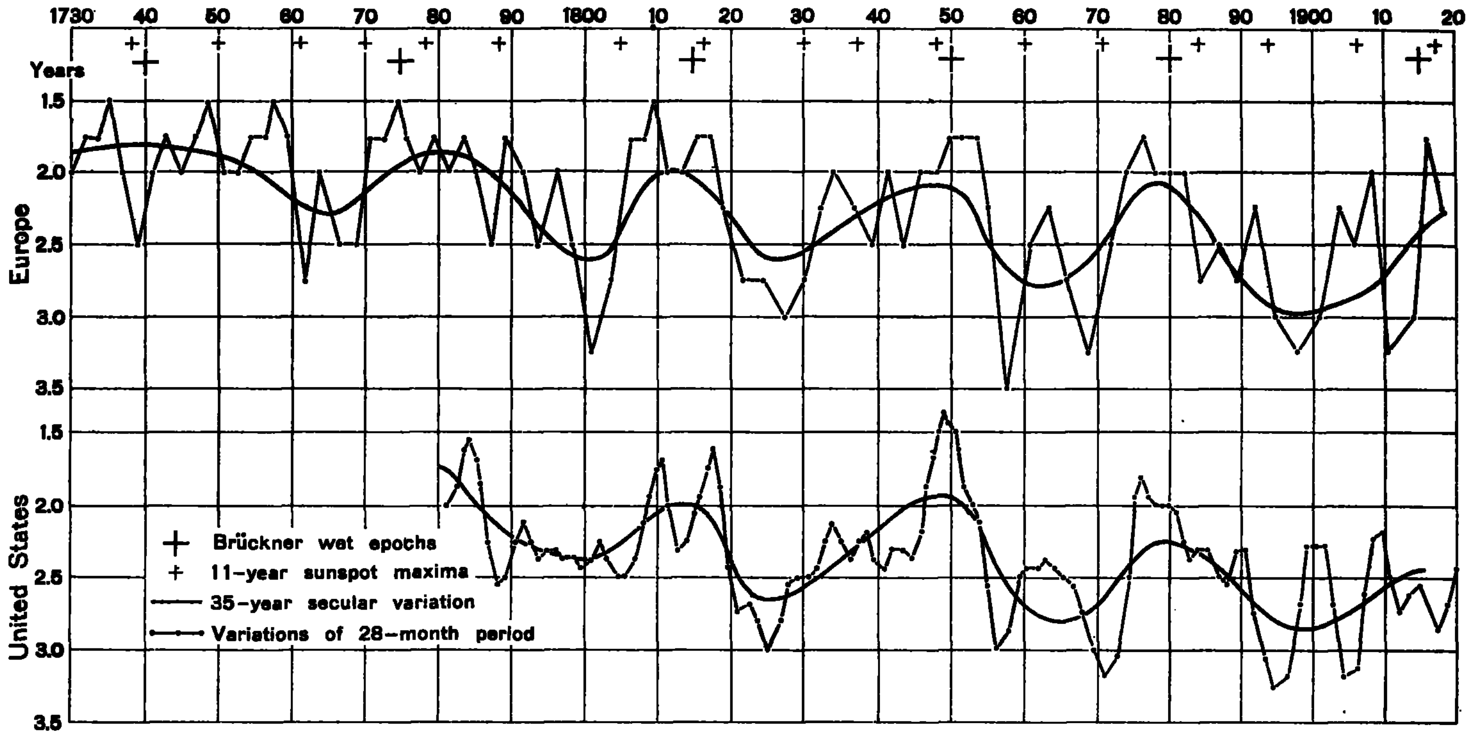

FIG. 3.—Curves representing the variation in the length of the 28-month period derived from the epochs of temperature for Europe and the United States (Table 1). The heavy smooth curve represents the 36-year variation in the length of the period

mean deviation is 0.39 year, and 77 per cent of these differences are between +0.50 and −0.50 year.

European pressure data for the southwestern portion, including Spain, France, and Italy, have likewise been studied, and it is found that epochs of low pressure in southwestern Europe (Table 1) usually precede, on an average by about 0.28 year, the epochs of low temperature over northern Europe, each epoch of low temperature being associated with a corresponding epoch of low pressure, and vice versa. This relation illustrates the well-known fact that, irrespective of the time unit employed, regions with negative pressure anomaly are associated with areas to the northward having negative temperature anomaly, and, on the other hand, areas with positive pressure anomaly have abnormally high temperatures to the northward. This is obviously due to the effect of an increased frequency of northerly winds in the first case and of southerly winds in the second.

The table below gives the frequencies of occurrence of the various differences between these epochs (pressure epochs minus temperature epochs), from which it appears that for Europe the mode is slightly greater than 0.C, and the mean deviation 0.44 year.

Differences, pressure—temperature		
Years	Frequencies	
	Europe	United States
−1.00	1	1
−0.75	0	1
−0.50	5	2
−0.25	10	5
0.00	17	19
+0.25	14	11
+0.50	13	14
+0.75	8	6
+1.00	8	3
+1.25	4	----------

The epochs of minimum temperature in the United States average 0.22 year later than the epochs of minimum pressure in Europe. The mean deviation of the phase differences is 0.34 year. The frequencies are shown in the table above.

Advantage was taken of this close relation between pressure and temperature in the determination of the epochs of maxima and minima of these two elements. It was early apparent that for each maximum or minimum of temperature there existed a corresponding phase of pressure, and where doubt existed as to the assignment of any particular epoch of temperature reference was made to the pressure curve for confirmation or change. In fact the pressure curve for southwestern Europe exhibits the short period with even greater regularity than the temperature curve for northern Europe.

Secular variations in the length of the period.—The phase intervals in Table 1 are plotted in Figure 3 opposite the midyear of the interval, and the continuous curve joining them represents the varying length of the period since 1730. The curve for the United States represents both maximum and minimum phase intervals smoothed by the formula $(a+2b+c)\div 4$. The Brückner wet epochs and the epochs of sun spot maxima are shown on the diagram. The 36-year secular variation in the length of the period is shown by the heavy smooth curve.

Inspection of this diagram discloses a tendency to minimum lengths of the period about 1750, 1780, 1815, 1850, 1880, and 1915, and to maximum lengths at the intermediate dates. These dates when the period is short coincide with the Brückner epochs of maximum precipitation in the interior of the United States and Europe. In other words, when the period length is 2 years or less, excessive precipitation over a period of years occurs, and when the length is 3 years or more a deficiency in precipitation occurs. A relation analogous to this was deduced by the writer from a study of the sun spot period. He showed (Astro. Jour., 1905) that a shortening of the period to 10 years or less preceded by about 10 years the Brückner epochs of excessive rainfall, and vice versa, a length of the period above the average, as 12 to 14 years, preceded epochs of deficiency of rainfall.

The results obtained by Brooks and Clayton, mentioned above, find their explanation in this variation in the length of the period. The two series of alternating winters referred to by Brooks were around 1815 and 1880, when the length of the period was about 2 years. Clayton's 25-month period likewise occurred around 1880. When the period lengthened to 2¾ years these features disappeared.

A secondary 11-year variation in the length of the period is indicated by a shortening of the period soon after the epochs of sun spot maxima, but the range of variation is less than that of the 36-year period.

The European temperature epochs show, furthermore, a tendency to a long secular increase in the length of the period from 2 years or less about 1750 to 2.5 years or more at the present time. This is consistent with the abnormally high levels of European rivers and lakes about the middle of the eighteenth century. The Caspian Sea was also much higher at that time than it has been at any subsequent time. This long-period variation is probably due to the 300-year cycle which the writer has shown elsewhere (Astro. Jour. 1905) to underlie the variations of the 36-year period.

Epochs of pressure and temperature at Batavia.—The observations at Batavia were studied in considerable detail. Three means per year were employed, each 4-month mean receiving an appropriate correction for annual variation. Curves were drawn for four elements, viz, pressure, mean maximum temperature, mean minimum temperature and rainfall, the latter element being expressed in the form of a frequency of occurrence, using the number of months with deficient precipitation in each 12-month period centered on January and July 1. By comparison of these four curves it was possible to determine definitive epochs for this place since 1866. The epochs of maximum and minimum pressure coincide closely with the epochs of minimum and maximum frequency of rainfall respectively. The epochs of the mean maximum temperature follow similar phases of the pressure by about 4 months as an average. The epochs of the mean minimum temperature follow by a few months the epochs of the mean maximum temperature. Table 2 gives the epochs of pressure and mean maximum temperature for this place. The fluctuations of pressure are entirely representative of the fluctuations for the whole Indo-Oceanic region, including India, Australia, and Mauritius.

TABLE 2.—*Epochs of maximum and minimum pressure and temperature at Batavia*

Pressure		Mean maximum temperature	
Maximum	Minimum	Maximum	Minimum
	1867.2		1867.5
1868.5	1870.2	1868.7	1870.7
1871.5	1872.8	1871.5	1872.2
1873.8	1875.0	1873.8	1874.7
1875.5	1876.3	1875.7	1877.0
1877.5	1878.7	1877.7	1879.5
1879.4	1880.0	1880.0	1880.7
1881.2	1882.3	1881.8	1882.5
1883.7	1884.5	1883.5	1885.0
1885.7	1887.2	1885.7	1887.7
1888.7	1890.0	1889.0	1890.7
1891.5	1892.5	1891.8	1893.8
1894.2	1895.5	1894.7	1895.5
1896.7	1898.5	1897.2	1899.0
1899.8	1901.3	1900.5	1901.5
1902.8	1904.2	1903.0	1904.6
1905.5	1906.8	1906.0	1907.0
1908.5	1910.2	1908.8	1910.2
1911.7	1913.2	1912.2	1913.0
1914.5	1916.7	1914.8	1916.8
1917.5		1917.5	

Epochs of pressure and temperature at Portland, Oreg.—It is found that pressure curves do not show this period as satisfactorily as the temperature curves except in certain regions where, in all but the winter season, cyclonic disturbances are relatively infrequent and the pressure relatively high. Such regions comprise the States of Oregon and Washington, southwestern Europe, particularly the Iberian Peninsula, and the Azores and Madeira. These regions of Europe and the United States have somewhat similar meteorological characteristics in that the ridge of subtropical high pressure overlies these western portions of the two continents. The pressure curves at Portland and San Fernando show how plainly evident the short period appears on casual inspection. Certain tropical regions also, as southern India and northern Australia, yield pressure curves in which the short period is clearly obvious.

The curve of pressure variations at Portland, Oreg. (fig. 1), shows the short period satisfactorily. As a rule, the epochs of minimum pressure and maximum temperature (Table 3) coincide at this place. The pressure epochs may precede or follow the temperature epochs by as much as one-half or three-fourths year occasionally, but over a long series of years the average deviation is close to zero. The curve of mean maximum temperature at Portland shows a feature which is absent from the curve of minimum temperature, namely, the simultaneous occurrence of short intervals and small amplitudes, the amplitude being a direct function of the length of period. The mean length of the period about 1880 was 1.8 years, and the extreme range averaged 1°.5. In 1890 the mean length had increased to 2.9 years and the range to 3°.5. In 1910 the mean length was 2.5 years and the range 1°.5. In 1920 the length was 3.0 years and the range 3°.5. As previously stated, the amplitude by the method employed is reduced to about 68 per cent. These values of the range should therefore be increased by about one-half to obtain the true range.

TABLE 3.—*Epochs of pressure and temperature at Portland, Oreg.*

Pressure		Temperature	
Maximum	Minimum	Minimum	Maximum
1873.0	1874.2	1872.5	1874.2
1875.0	1876.0	1875.0	1876.0
1877.0	1878.0	1877.0	1877.5
1879.0	1879.7	1878.7	1879.5
1880.7	1882.0	1880.2	1881.2
1883.0	1884.2	1882.0	1883.2
1885.0	1886.0	1884.5	1885.5
1887.7	1889.5	1887.5	1889.0
1890.7	1891.5	1890.2	1891.7
1893.0	1894.2	1893.5	1894.7
1895.7	1896.7	1896.0	1897.5
1898.5	1900.0	1899.0	1900.0
1901.5	1902.5	1901.0	1902.5
1903.5	1904.7	1903.5	1905.0
1906.0	1907.0	1906.0	1907.2
1908.5	1909.5	1909.0	1910.2
1911.0	1912.5	1911.2	1912.2
1913.5	1915.2	1913.5	1915.0
1917.2	1919.0	1916.7	1918.0
1920.0	1921.0	1920.0	1921.0
1922.0	1923.0	1922.5	1924.0
1924.5			

Epochs of pressure for Greenland and Iceland.—Pressure data are available for Iceland since 1846 and for Greenland from 1842 to 1851 and since 1866. A curve of these variations in pressure at Jacobshavan, Greenland, since 1870 is shown in Figure 1. In general the epochs for Greenland and Iceland closely agree. Table 4 gives the epochs based on the Jacobshavn data except from 1850 to 1866, when they are derived from the Iceland data. Comparison with the epochs of minimum temperature in Europe shows a tendency for high pressure in the Arctic regions to precede low temperature in Europe by an average of 0.27 year. The mean deviation of the temperature lag from this average value is 0.28 year. This relation is entirely confirmatory of similar relations found by Brückner, Hann, and others for both cyclical and noncyclical variations.

TABLE 4.—*Epochs of maximum and minimum pressure in Greenland and Iceland*

Maximum	Minimum	Maximum	Minimum
1844.5	1845.5	1882.5	1884.0
1846.5	1847.5	1885.7	1886.7
1848.7	1849.5	1888.0	1890.0
1850.2	1851.0	1891.0	1891.5
1852.0	1852.5	1892.5	1894.2
1853.2	1854.5	1896.2	1898.5
1856.0	1857.5	1899.7	1901.0
1859.5	1860.0	1902.2	1903.0
1861.7	1863.0	1904.5	1905.5
1865.0	1866.2	1907.2	1908.5
1867.0	1868.0	1910.0	1911.5
1869.7	1872.0	1912.5	1913.5
1872.7	1874.2	1915.5	1916.5
1875.0	1875.7	1917.5	1918.2
1876.7	1877.7	1919.2	1920.5
1878.5	1880.0	1922.5	....
1881.0	1882.0	....	....

Epochs of pressure variability at St. Louis.—The mean interdiurnal variability of pressure at St. Louis has been computed for each month from 1888 to 1923, inclusive. Both 8 a. m. and 8 p. m. observations have been employed for this purpose. A curve showing variations in this element is shown in Figure 1, and the 28-month epochs of maxima and minima are given in Table 5. The epochs of maximum and minimum variability precede, respectively, the epochs of minimum and maximum temperature by an average of 0.30 year. Correlation of these variations with the temperature at Winnipeg in winter yields −.57, while with the temperature at New Orleans the correlation is +.40. The inference is that large day to day changes in pressure at St. Louis are associated with temperature below normal to the northward, and temperature above normal to the southward. The latitudinal temperature gradient at such times is increased and areas of high and low pressure move more rapidly. This is shown by the correlation between the pressure variability and the velocity of movement of low pressure areas in the United States during the months December to March, which amounts to +.45.

TABLE 5.—*Pressure variability at St. Louis*

Maximum	Minimum	Maximum	Minimum
....	1889.0	1906.5	1907.5
1890.2	1891.7	1908.7	1910.5
1893.5	1894.7	1911.7	1913.5
1896.0	1897.5	1914.5	1915.2
1898.5	1900.0	1917.0	1919.0
1901.0	1902.2	1920.0	1921.0
1904.0	1905.2	1923.0	....

Velocity of movement of areas of low pressure.—The mean velocity of low-pressure areas over the United States from 1872 is shown in Figure 1. The curve is inverted to show the inverse correlation between temperature, particularly in high latitudes, and the mean velocities of storms. The curve of temperature at Edmonton, for example, shows a high correlation with the velocity curve. Table 6 gives the epochs of the short cycle.

TABLE 6.—*Epochs of maximum and minimum velocity of Lows*

Maximum	Minimum	Maximum	Minimum
....	1873.0	1888.2	1889.5
1875.2	1876.0	1890.2	1891.2
1877.0	1878.0	1893.0	1894.7
1879.5	1880.5	1896.5	1997.5
1881.5	1882.7	1898.5	1899.5
1883.5	1884.0	1900.5	1901.2
1885.0	1886.2	1903.0	....

Variations of rainfall and lake levels and correlation with pressure and temperature.—The close relation between pressure and temperature in Europe, shown above, is clearly due to the intimate relation between wind direction and temperature. An increase in the northerly component at the surface is associated with pressure below normal over southern Europe, and vice versa. With rainfall, however, the relation is more complicated. An excess of rainfall, for example, involves more than one factor. There must be (1) an excess of water vapor in the atmosphere which is usually due to the prevalence of winds with a southerly component and from a water surface; (2) the occurrence of cyclonic conditions which are favorable to the general ascent of vapor-laden air and consequent dynamic cooling and condensation. These two factors do not usually coexist in northern Europe, since the prevalence of winds from the Atlantic is favored by high pressure over central and southern Europe and low pressure over Iceland; in other words, anticyclonic conditions exist over Europe, which are unfavorable to the occurrence of general ascent of air. Although the vapor content of the air may be high, the absence of the requisite condition for its abundant condensation results in only moderate rainfall.

These considerations largely account for the fact that rainfall data do not exhibit the short period as regularly and definitely as pressure and temperature. Statistical analyses, moreover, have shown that rainfall data are more nearly allied to perfectly fortuitous data than other elements, and for this reason it is probable that rainfall, the importance of which is greater than that of temperature, will never be as satisfactorily forecast as will temperature.

Epochs of maxima and minima of rainfall at Upsala and of the level of Lake Malar have been assigned and the lake epochs are given in Table 7. The epochs of lake level follow by about 0.2 year the epochs of rainfall. As a rule, the epochs of maximum lake level usually precede slightly the epochs of minimum pressure, the average interval being 0.10 year; but at times, notably around 1845 and 1880, they are nearly coincident with or even follow the pressure epochs. The rainfall curve shows numerous secondary maxima and minima, due partly, as above stated, to the presence of periods shorter than the 28-month period and partly to merely minor accidental features or imperfectly eliminated or abnormal annual variation. For this reason Wallén derived a period of only 24 months for the rainfall as against 30 months for the lake levels. The epochs of lake level assigned by me differ from Wallén's in some respects. I have inserted three additional maxima, only slightly developed, and combined two of his adjacent maxima into one. These changes reduce the mean length of the period to 28 months, which is also the mode for this lake, as before stated.

TABLE 7.—*Epochs of high and low water in Lake Malar*

High	Low	High	Low
1825.0	1826.7	1870.0	1871.8
1828.0	1830.0	1872.9	1874.5
1831.0	1832.8	1875.0	1875.7
1834.0	1835.6	1877.5	1878.5
1836.5	1837.8	1879.5	1880.4
1838.7	1840.0	1881.8	1883.0
1841.6	1843.0	1883.8	1884.7
1843.5	1844.1	1885.7	1887.0
1844.7	1845.5	1888.5	1889.5
1846.2	1847.1	1890.5	1892.8
1848.5	1849.5	1894.0	1895.0
1850.2	1851.0	1896.0	1897.5
1851.7	1852.3	1898.7	1899.9
1853.2	1854.0	1900.8	1901.9
1855.2	1856.0	1903.5	1905.0
1857.5	1859.5	1906.0	1907.0
1860.6	1862.0	1907.9	1909.0
1863.5	1865.5	1910.5	----------
1867.0	1868.7		

For reasons previously stated, the lake variations are regarded as more suitable for the determination of the epochs of the short period than rainfall variations, and these epochs are therefore regarded as representative of the rainfall variations in Sweden. In other countries, however, an entirely different régime may prevail.

The rainfall for Oregon varies in a fairly regular manner in the 28-month period. Curves showing the rainfall variations for Portland and Roseburg are shown on Figure 1. The epochs of minimum and maximum rainfall for Oregon coincide closely with the epochs of maximum and minimum pressure, respectively, at Portland, there being an average lag or retardation of 0.13 year in the rainfall epochs. This relation is similar to the result found for Sweden, and also to that for Batavia.

Correlations of temperature in the Mississippi Valley and lag between northern and southern stations.—The epochs of the short period for St. Paul, St. Louis, Memphis, Vicksburg, and New Orleans have been derived, and it is found that there is an average lag of 0.19 year from St. Paul to St. Louis, and a lag of 0.37 year between St. Paul and New Orleans. From St. Louis to New Orleans there is a lag of 0.20 year. The short-period fluctuations thus occur later in low latitudes.

This is confirmed by the correlation coefficients for St. Paul and Vicksburg, employing 12-month means, 6 months apart. With simultaneous values the coefficient is +0.34. With Vicksburg temperatures, 6 months previously, the coefficient is −0.26, and a year previous, −0.38. On the other hand, with Vicksburg temperatures 6 months later the coefficient is +0.48, or considerably greater than for simultaneous temperatures.

The inference is that the waves of high and low temperature travel equatorward over North America. The amplitude at southern stations is also much less than that at northern stations. Whether the lag at southern stations is a universal phenomenon is for further investigation to determine, but it may be stated that Clayton's [26] researches lead him to a similar inference.

VI. THE 28-MONTH PERIOD IN SOLAR PHENOMENA

Bigelow and Lockyer endeavored to correlate variations in the frequency of solar prominences with meteorological variations. The prominence data, are, however, unsatisfactory, owing to the necessary limitation of the observations to the solar limb and to the lack of homogeneity in the data by different observers.

Variation in mean latitude of sun spots.—The variations of the mean solar latitude of the entire spotted area as measured by the Greenwich Observatory since 1875 comprise a homogeneous body of data which exhibits the short period very satisfactorily. The data published are means during successive synodic rotation periods of the sun, employing the period deduced by Carrington, 27.27 days. I have averaged the data by groups approximating the periods January 1 to June 30 and July 1 to December 31, to obtain two means per year. These are shown in Table 8. The plus signs represent north latitude and the minus signs south latitude. Column 1 is the first half and column 2 the second half of the year.

TABLE 8.—*Mean solar latitude of entire spotted area, in degrees*

	(1) January-June	(2) July-December		(1) January-June	(2) July-December
	°	°		°	°
1874	----------	−3.7	1900	−1.6	−1.6
1875	+1.6	+1.4	1901	−1.4	−2.4
1876	−4.1	−4.6	1902	+16.0	+6.0
1877	−2.3	−1.6	1903	+4.7	−4.0
1878	+7.2	+6.2	1904	+0.7	+1.3
1879	−10.3	−3.3	1905	−1.7	+3.3
1880	+9.4	+5.2	1906	+6.1	−0.1
1881	−2.7	+10.7	1907	−4.5	−2.0
1882	−5.7	+1.7	1908	−4.3	+2.0
1883	−5.6	−6.9	1909	−2.9	−3.3
1884	−4.0	+0.1	1910	−4.0	−5.6
1885	−4.6	−2.2	1911	−3.3	+0.0
1886	−7.1	−2.4	1912	−10.0	−4.5
1887	+2.3	−4.3	1913	+15.0	+6.6
1888	−4.4	−5.0	1914	+7.1	+3.1
1889	−1.5	−10.0	1915	+5.2	−0.7
1890	+2.1	−0.3	1916	+0.4	+9.6
1891	+2.4	+9.4	1917	−0.3	+0.5
1892	−3.7	−2.8	1918	+1.9	−0.5
1893	−6.1	−2.0	1919	+0.6	−2.4
1894	−4.3	−2.3	1920	−0.3	−3.1
1895	+3.3	+3.2	1921	−1.0	+3.0
1896	−4.1	−6.0	1922	+4.0	−3.6
1897	−3.5	+0.1	1923	----------	----------
1898	−6.0	−4.7	1924	----------	----------
1899	−7.8	−6.3			

A sudden shifting of the excess of spots from southern to high northern latitudes occurs shortly after the spot minimum, resulting in a pronounced 11-year variation in the mean latitude. I have eliminated this 11-year variation by taking residuals from a smooth curve representing the secular trend, and a plot of these residuals, slightly smoothed is shown in Figure 1. This curve shows the pure 28-month variation and may be readily compared with meteorological curves. The epochs of this variation are given in Table 9, representing the mean dates when excesses of spots occurred alternately in the two hemispheres. The average length of the period between 1875 and 1924 is 2.55 years, varying from 2.2 years in 1880 to 2.9 years in the late nineties and then decreasing to 2.5 years about 1915 to 1920.

TABLE 9.—*Epochs of solar spot variation in latitude*

North	South	North	South
1875.7	1877.0	1900.2	1901.5
1878.5	1879.2	1902.5	1903.7
1880.2	1881.2	1906.0	1907.5
1882.0	1883.5	1908.7	1910.2
1884.7	1886.0	1911.7	1912.5
1887.2	1888.2	1914.0	1915.5
1889.2	1889.7	1916.7	1917.5
1891.7	1893.2	1919.0	1920.5
1895.5	1896.5	1922.0	1922.7
1897.7	1899.2	1924.2	----------

Employing the method of correlation illustrated in Section II by the data at St. Paul, the following table gives for the solar data plotted in figure 1 the results of correlating values of the same data separated by successive time units, in this case half-yearly intervals.

[26] Clayton, H. H., World Weather, 1924, p. 269.

Years	r	years	r
0.0	+1.00	3.0	-.18
0.5	+.36	3.5	-.14
1.0	-.76	4.0	-.11
1.5	-.52	4.5	+.49
2.0	+.46	5.0	+.19
2.5	+.48		

The maximum positive coefficients are at 2.25 and 4.5 indicating a length of period of about 2.25 years.

Variation in Wolfer's relative sun-spot numbers.—To extend the solar epochs back from 1875, I have plotted the Wolfer sun-spot numbers (smoothed) beginning with 1750, and also the 6-month means, January to June and July to December, inclusive. Through the minor fluctuations superposed upon the 11-year period a smooth curve was then drawn forming the primary 11-year wave. Thus it was possible to determine from these curves, which facilitated elimination of the secular trend, the epochs of secondary spot maxima and minima, and while these epochs are not wholly satisfactory, owing to the approximate character of the data, especially in earlier years, yet it is clear that the 28-month period is present in these secondary fluctuations and has persisted since 1750, and that its length has varied synchronously with that of the meteorological period. These epochs of spot maxima are given in Table 1.

The following table gives the frequency of the intervals, which have varied between 1.5 and 3.5 years.

Interval	Cases	Interval	Cases
1.50	7	2.75	5
1.75	12	3.00	7
2.00	14	3.25	1
2.25	17	3.50	3
2.50	10		

The mean length is 2.27 years, while the mode is approximately 2.15 years. The distribution is somewhat unsymmetrical and similar to that of the 11-year period. The mean deviation is 0.38 year and the mean variability 0.41 year.

VII. CORRELATION OF SOLAR AND METEOROLOGICAL DATA

When a curve of the solar variations in latitude is compared with a curve of terrestrial data, as, for example, the temperature at St. Paul, it is apparent that each epoch of low temperature is preceded by a corresponding epoch of spot excess in the Northern Hemisphere, the average interval of time intervening, or the time-lag, being about 1.00 year. This time-lag, being a direct function of the length of the period, varies, being about three-fourths year in 1880 and 1915 when the period length is short, and 1⅓ years in 1895. (Cf. fig. 4.) Correlating the solar and temperature data for the period 1875-1923 for simultaneous values and also for successive lags, varying by half-yearly intervals, in the temperature, the following results are obtained. For simultaneous values the result is set opposite zero in the table; shifting the temperature curve to the left by successive half-yearly intervals the results are as shown:

Years	r	Years	r
0.0	+.40	3.0	-.63
0.5	-.51	3.5	+.15
1.0	-.56	4.0	+.60
1.5	+.26	4.5	+.31
2.0	+.60	5.0	-.50
2.5	+.15	5.5	-.31

This table shows that the phases of the two curves come into approximate conjunction or opposition with each other, as the temperature curve is shifted by successive half-yearly intervals, on an average of about every 2¼ years.

When the sunspot epochs from 1750 are compared with the European epochs of low pressure (fig. 2), it is clear that there is a remarkable correspondence in the two series. The solar epochs precede the meteorological epochs by an average interval of 1.6 years. The time interval varies widely in a 36-year cycle, ranging from near coincidence when both phase intervals are short, or around 1750, 1775, 1815, 1845, 1880, to 3 years or more when the phase intervals are long. Here, as elsewhere, the time interval or lag is a direct function of the length of the periods.

Since 1875, when the latitude variation data become available, these epochs are to be preferred to the epochs of spot maxima as representative solar data to be correlated with the meteorological epochs. The epochs of excess spottedness in the Northern Hemisphere precede the epochs of low pressure in Spain by an average of 0.9 year, and the epochs of high pressure at Portland, Oreg., by about 0.6 year.

VIII. GRAPHICAL EVALUATION OF THE LENGTH OF A VARIABLE PERIOD

Figure 4 is a diagrammatic representation of a periodicity tabulation of the mean latitude of spots with only the secular variation eliminated. The dates at the left are given at intervals of 2½ years beginning with 1872.5. The vertical lines are one year apart. Beginning with each date 1872.5, 1875.0, etc., the data are plotted for five years so that the second half of each curve is identical with the first half of the next curve. The epochs of maximum north and south latitude are indicated on the line of zero latitude for each section and curves of best fit have been drawn through these points. These curves of best fit incline to the left until 1885–1890, indicating a length of period less than 2½ years, thereafter to the right, indicating a mean length greater than 2½ years. In recent years the line inclines slightly to the left again. The curve of best fit for the minima has the same general course as the curve for the maxima, but their distance apart is least around 1880 when the slope of the curves indicates that the length of the period is shortest. The spread gradually increases until about 1895, then decreases to about 1915–1920.

A further feature is an 11-year variation in the length of the period, superposed on the long 36-year variation. Shortly after the epochs of spot minima, indicated on the diagram, the maxima of the curves are far to the left of the line of best fit, while on the next recurrence the maximum has shifted over to the right, indicating an excessively long interval at that time.

This graphical scheme is an extension of the familiar numerical periodicity tabulation combined with the well-known shift of phase when the length of the period differs somewhat from the time covered by a single row of the tabulation.[27]

The epochs of minimum temperature for the United States are shown plotted on Figure 4 with the curve of best fit drawn. This curve follows the curve of north latitude of spots by about one year as an average, but the spread of the two curves varies directly with the length of the period in a 36-year cycle.

[27] Cf. Brunt. Combination of Observations, p. 199.

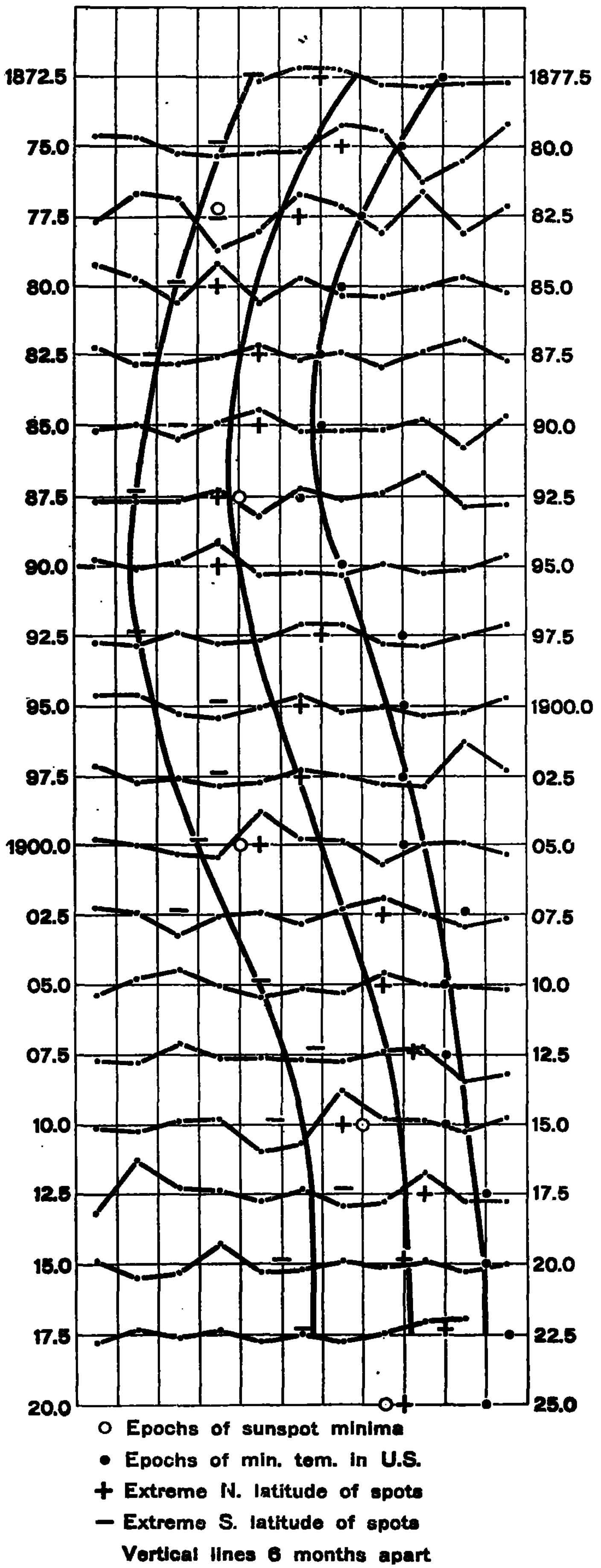

FIG. 4.—Graphical periodicity tabulation of mean latitude of sun spots, with a fundamental interval of 30 months. The horizontal curves comprise a time interval of 60 months. The vertical curves are curves of best fit drawn through the points on the axis of each horizontal curve, which represent the locations of the phases of maxima and minima of the 28-month period.

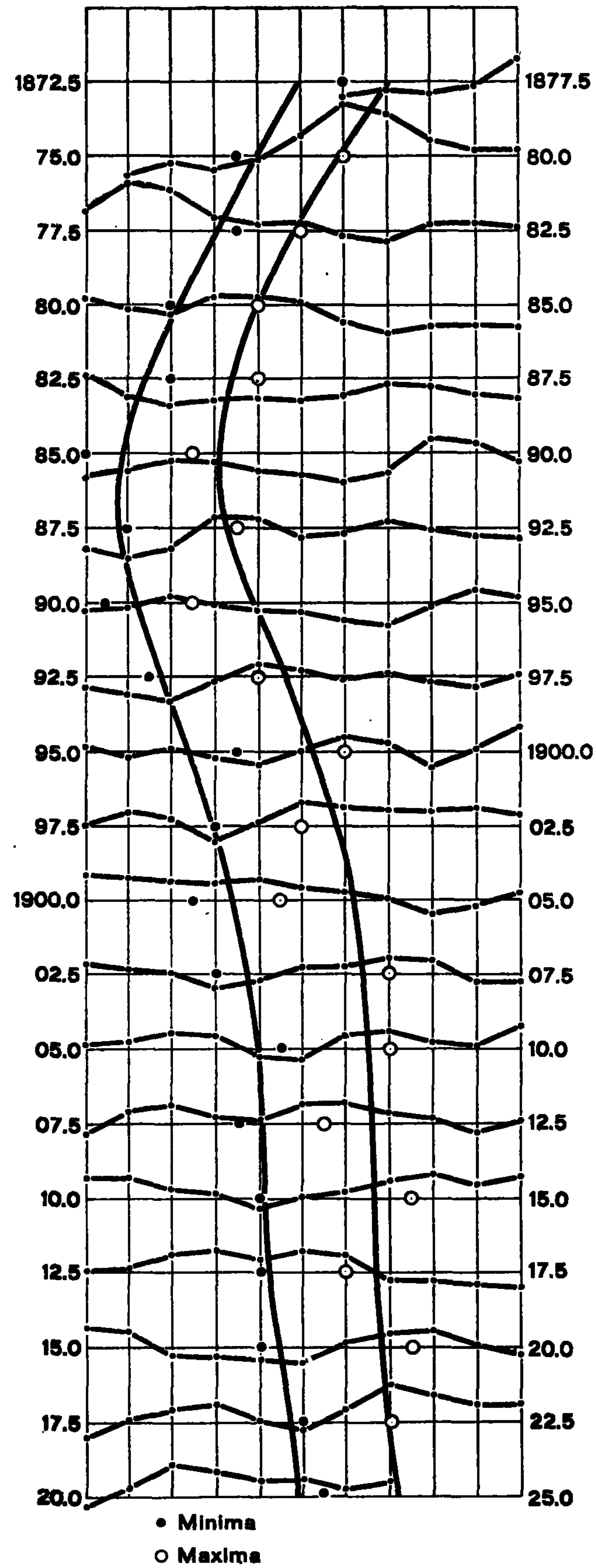

FIG. 5.—Graphical periodicity tabulation of combined temperatures for six stations in Minnesota, North Dakota, and Manitoba.

Figure 5 is a diagram of the composite curve of temperature from 1872. The deviations of the maximum and minimum phases on either side of the curve of best fit are, as stated above, partly due to the 11-year variation and partly due to other causes, among which are certain uneliminated short-period and accidental features of the data.

IX. CONCLUSION

The object of the investigation has been, as previously stated, the determination of the epochs of maxima and minima of the short period for restricted areas, and the systematic variations in its length. This has been accomplished satisfactorily by consideration of data for temperature and pressure in Europe and the United States only, where long homogeneous records are available.

The variations of rainfall have been given only secondary and partial consideration, since examination of data from the relatively few stations from which the epochs of temperature and pressure were derived have led to indefinite results for rainfall, owing to the local and fortuitous features of this element. Records from a large number of stations must be averaged to yield satisfactory results.

Large regions of the globe including Africa, Asia, and most of the Southern Hemisphere have been left for future work. The regions adjacent to the eastern Indian Ocean, including Australia, have been studied rather intensively so that the epochs for this section, well represented by Batavia, can be said to be definitively established.

The amplitude of the period has not been studied to any extent, since its exact determination is a complicated process which must be left for future development. From inspection of the curves certain qualitative conclusions may be drawn. It is probable that the amplitude varies directly with the wave length, as illustrated by the amplitude of the temperature wave at Portland (Section V); also that the amplitude is smaller than that of variations of longer period, as the 7-year period.

Regarding the correlation found to exist between solar and meteorological data, the paper presents simply the findings of the investigation without adequate physical reason for such relationship, which appears to be of a causal nature. In this, as in many other cases, theory must wait upon observation and there is much to be done before a satisfactory basis for induction can be said to exist.

DISCUSSION

By CHARLES F. MARVIN

Convinced of the great importance of serious and searching investigations of solar and terrestrial correlations and of the laws of sequence of weather conditions, the writer has encouraged and supported such work at the central office of the Weather Bureau to the fullest extent permitted by the limited personnel and funds available.

While the present paper represents the results of only a recent investigation, nevertheless these matters have been a subject of unofficial work and study by Mr. Clough for fully 20 years, all of which constitutes a substantial foundation and background for the present contribution.

Unwilling to publish in the MONTHLY WEATHER REVIEW a paper on the illusive question of periodicities by a member of our staff which could not command the approval, at least in general and tentative terms, of myself and others, this paper has received more than the usual critical examination, both by the committee on scientific papers and myself, with the result that it is believed Mr. Clough supports his findings by a substantial array of proofs derived directly from (1) a large and acceptable body of meteorological data, (2) reference to a series of seemingly incontrovertible statistical criteria, and (3) solar and terrestrial correlation of material significance. The study has been purely an inductive one and wholly lacks suggestions as to the physical causation of the findings. The task of the critic, therefore, is, of course, to successfully refute the argument, to interpret the evidence in other terms, and to show that the findings as a whole or in part are not the facts they are represented to be. If disproofs are not forthcoming in adequate form, we must conclude that Mr. Clough's views constitute an important contribution to the laws of sequence of weather conditions.

Unfortunately, readers but partly acquainted with the literature and the technique of the subject of weather periodicities will probably find it difficult to pick out the essence of Mr. Clough's findings from the mass of details which he has deemed it necessary to present in order to meet and forestall probable criticism and exceptions.

His claims are:

(1) The sun-spot cycle is a prototype of other solar and terrestrial periodicities, many of them being obscure and unknown.

(2) The seemingly erratic and irregular changes in lengths of such of these periods as some recognize, are due only in part to accidental causes, errors of determination, etc. In addition, some as yet unknown but dominant physical influences cause the lengths to change in a systematic manner.

(3) That there is a rhythmic response and a corresponding correlation between solar and terrestrial periodic phenomena.

Assuming that we have fairly stated Mr. Clough's major findings, let us examine critically some of the evidence and proofs thereof.

Raw material and smoothing formula.—With very few exceptions, the statistical data employed were in the form of 6-month means smoothed by the formula $(a+b)\div 2$, which gives a result exactly the same as 12-month means taken at 6-month intervals. While the amount of the smoothing in this case is very slight, nevertheless it must be recognized that the application of any smoothing formula to a sequence of numbers *tends to create consecutive correlation where none previously existed.* Furthermore, where obscure correlation of a periodic character already exists the smoothing tends to reduce or efface the amplitude and to shift or alter in a more or less unassignable way important phase relations.

Suppose a, b, c, d, e, f, etc., represent an irregular sequence of perfectly unrelated numbers. Let this sequence be smoothed by any such formula as, say $(a+2b+c)\div 4$, etc. There is at once created an implied relation of an obscure character between each derived value and those immediately contiguous thereto.

These considerations apply to all such results as are represented by the curves in Figures 1 and 2 of Mr. Clough's paper. However, the practice of using smoothing formulæ in cases of this character is all but universal, and Mr. Clough's use of them seems to be as fair and conscientious as that of any other investigator. Accordingly, we find no adequate ground for the rejection as a whole of the dates of maximum and minimum phases